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THE
CAMBRIDGE SERIES
FOR SCHOOLS
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TRAINING COLLEGES

GEOMETRICAL DRAWING

PART I

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GEOMETRICAL DRAWING

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GEOMETRICAL DRAWING

WITH NOTES AND EXAMPLES

BY

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PART I

Plane and Elementary Solid

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PREFACE.

THIS book is intended for Candidates reading Geometrical Drawing for the Science and Art Department, South Kensington, Elementary Stage, for entrance to Sandhurst and Woolwich, and for the Cambridge Preliminary and Junior Local Examinations.

Permission has been obtained from the Assistant Controller of Her Majesty's Stationery Office, and from the Syndicate of the Cambridge Local Examinations, to republish questions set as examples in this book, provided the source from which they are taken is duly acknowledged, and that mention is made of the fact that permission has been obtained.

The Civil Service Commissioners are in no way responsible for the republication of the questions.

Part I contains Plane Geometrical Drawing and Solid Geometry so far as it is included in the Syllabus, Geometrical Drawing (Art), Subject 1 A, page 231. Science and Art Directory 1899.

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In Part II is added a course of instruction in Solid Geometry suited to the Elementary Stage of Science Subjects as given on page 78 of the above Directory.

The works of the following authors have been consulted: Henry Angel, John Carroll, R. Harris, D. A. Low, James Martin, J. B. Millar, E. C. Plant, J. S. Rawle, J. H. Spanton, S. H. Wells, S. H. Winter, and the Text-books used at Sandhurst and Woolwich.

W. H. B.

December, 1899.

Science and Art examples are marked (S).
Civil Service Commission ,, ,, (A).
Cambridge Preliminary ,, ,, (P).
Cambridge Junior Locals ,, ,, (C).

Reference is made to any article and the figure corresponding to it by its number in a bracket, i.e. figure 120, *as* (120).

CONTENTS.

INTRODUCTION . . PAGE xiii

CHAPTER I.

SIMPLE CONSTRUCTIONS . . PAGE 1

1. To bisect a given angle. 2. To bisect a given straight line. 3. To draw a perpendicular to a straight line from a point within it and (4) from a point without it. 5. To construct an equilateral triangle on a given base. 6. To construct an isosceles triangle of given sides. 7. To construct a scalene triangle of given sides. 8. To construct a triangle having given two sides and included angle. 9. To describe a square upon a given base. 10. To describe a rhombus having given a diagonal and one side. 11. To describe a rhombus having given an angle and one side. 12. To describe a parallelogram having given a diagonal and its sides. 13. To describe a parallelogram having given an angle and its sides. 14. To construct a rectangle having given its sides. 15. Through a given point to draw a parallel to a given straight line. 16. To describe a square its diagonal being given. 17. To describe a rhombus the diagonals being given. 18. To construct any rectilineal figure the sides and diagonals being given.

CHAPTER II.

MEASUREMENT OF ANGLES . PAGE 10

19. To draw a perpendicular to a straight line from the end of it without producing the line, and (20) to draw it when the given point is without the line. 21. To construct an angle 60°. 22. The angle 30°. 23. The angle 15°. 24. The angle 45°. 25. The angle 75°.

26. The angle 105°. 27. To describe a right-angled triangle when the hypotenuse and one side are given. 28. To describe a rectangle when the diagonal and one side are given. 29. To make an angle equal to a given angle at a given point in a given straight line. 30. To draw a tangent to a given circle. 31. To draw tangents to a given circle from a given point without it. 32. To describe a segment of a circle on a given base to contain a given angle. 33. To cut off from a given circle a segment to contain a given angle. 34. To find the centre of a given circle. 35. To find the straight line which bisects the angle between two straight lines that do not meet in the plane of the paper [see also 156]. 36. To find a straight line passing through a given point, and the intersection of two straight lines, that do not meet in the plane of the paper.

CHAPTER III.

PROPORTION OF STRAIGHT LINES . PAGE 20

37. To divide a straight line into any number of equal parts. 38. To divide a straight line into parts in any given proportion. 39. To find a fourth proportional (greater) to three given straight lines, with a note on third proportional. 40. To find a fourth (or third) proportional less. 41. To divide a straight line proportionally to another. 42. To find a mean proportional to two given straight lines.

CHAPTER IV.

EXTREME AND MEAN RATIO . PAGE 24

43. To divide a straight line in extreme and mean ratio. 44. To construct a triangle with angles 72°, 72°, 36° [note on angle 18°]. 45. To inscribe a regular pentagon within a given circle. 46. To describe a regular pentagon upon a given base.

CHAPTER V.

INSCRIBED AND CIRCUMSCRIBING CIRCLES PAGE 28

47. To inscribe a circle in a given triangle. 48. To circumscribe a given triangle by a circle. 49. To describe a circle escribed to a given triangle. 50. To inscribe a circle in a quadrilateral under certain conditions.

CHAPTER VI.

ON THE CONSTRUCTION OF TRIANGLES PAGE 30

51. Given two sides and angle opposite one of them. Case I. 52. Case II. of the same proposition. 53. Given the altitude and angles at the base. 54. Given the base and vertical angle of an isosceles triangle. 55. Given the base, and angles in given proportion. 56. Given the base, vertical angle, and another side. 57. Given the base, a base angle, and sum of sides. 58. Given the base, a base angle, and difference of sides. 59. Given the base, and angles in given proportion. 60. Given the sum of the sides, and two angles. 61. Given the sum and proportion of the sides. 62. Given the base, vertical angle, and the proportion of the other two sides. 63. Given the three medians. 64. Given the altitude, base, and vertical angle.

CHAPTER VII.

ON THE CONTACT OF CIRCLES AND STRAIGHT LINES . . . . PAGE 36

65. To draw two external tangents to two given circles. 66. To draw two internal tangents to two given circles. 67. To draw a circle of given radius to touch two given circles externally. 68. To be touched internally by them. 69. To be touched externally by one, internally by the other. 70. To draw a circle of given radius to touch a given circle and a given straight line. 71. A circle of given radius to touch two given straight lines. 72. To draw a tangent to a given circle to make a given angle with another given tangent. 73. To draw a chord of a circle of given length to pass through a given point. 74. To draw a circle to touch a given circle and a given straight line at a given point. 75. To touch a given straight line, and a given circle at a given point. 76. To pass through two given points and to touch a given straight line. 77. To touch a given circle and pass through two given points. 78. To draw three circles with three given centres to touch each other. 79. To find on a given straight line a point from which we draw a tangent of given length to a given circle. 80. To draw a segment of a circle where the centre is not within the paper, but three points on it are known.

CHAPTER VIII.

SIMILAR FIGURES . . PAGE 46

81. To describe a triangle similar to a given one, having its sides parallel to those of the given triangle and an inscribed circle of given radius. 82. To describe the triangle when the circumscribing circle is of given radius. 83. Draw a circle to touch two given straight lines and pass through a given point. 84. To inscribe a series of circles between two straight lines and a given circle, each to touch the two given straight lines, and two of the circles. 85. To describe in a given triangle one similar to another. 86. To describe about a given triangle one similar to another. 87. To inscribe a square in a given triangle. 88. To inscribe a square in any regular figure. 89. To inscribe a square in a semicircle. 90. To inscribe a square in a given sector. 91. To inscribe an isosceles triangle in a given square, base given. 92. When the angle is given. 93. To inscribe a semicircle in a given isosceles triangle. 94. To inscribe a semicircle in a given sector. 95. To describe about a given circle a triangle the angles of which are given. 96. To inscribe a triangle the angles of which are given, in a given circle. 97. To place a straight line within a given angle, with its ends on the lines bounding the angle and parallel to another given straight line. 98. To describe within a regular figure another similar one having its sides parallel to those of the first, and of given length. 99. To describe about a regular figure another similar one, having its sides parallel to those of the first, and of given length. 100. To describe a figure on a given base similar to a given one. [To inscribe a circle in a given sector see 139 *b*, and note to example 136. Many other useful constructions will be found in the notes to examples.]

CHAPTER IX.

ON THE CONSTRUCTION OF REGULAR POLYGONS
PAGE 58

101. Heptagon in a circle. 102. Octagon in a circle. 103 and 104. Any polygon in a circle. 105. Any regular polygon on a given base. 106. Octagon on given base. 107. Regular polygon about a given circle. 108. Octagon in square.

CHAPTER X.

SCALES . . . PAGE 64

109. Plain Scale. 110. Diagonal Scale. 111 and 112. Scale of chords.

CHAPTER XI.

ON DRAWING FIGURES TO SCALE PAGE 72

113. By proportion. 114. By scale and by method of squares.

CHAPTER XII.

ON AREAS . . . PAGE 78

115. To reduce a rectilineal figure to a triangle of equal area. 116. To find a figure similar to a given one and bearing a given proportion to it in area. 117. To find a square equal in area to a given triangle. 118. To find a square equal in area to the sum of given squares. 119. To divide a circle into any number of annuli of equal area. 120. To divide a circle into any number of portions of equal area, and equal circumference. 121. To describe a figure equal in area to one given figure and similar to another. 122. To describe an equilateral triangle equal in area to a given triangle.

CHAPTER XIII.

ON SQUARE ROOT . . PAGE 88

123 to 126. To find the square root of a given number by four different methods.

CHAPTER XIV.

CONIC SECTIONS . . PAGE 92

127 to 129. To describe an ellipse. 130. To find the centre of a given ellipse also a tangent and normal. 131. The hyperbola. 132. The parabola.

CHAPTER XV.

MISCELLANEOUS . . PAGE 100

133. Trefoil of semicircles. 134. Quatrefoil. 135. Equilateral arch. 136. Spiral of semicircles. 137. Lancet arch. 138. Obtuse arch. 139. Foiled figures. 140. Elliptical arch. 141. An oval. 142. The cycloid. 143. The epicycloid. 144. The hypocycloid 145. The trochoid. 146. Involute of circle. 147. The spiral of Archimedes. 148. A spiral the eye of which is given. 149. Eye of Ionic volute. 150 to 153. Construction of figures. 154. A four-centred arch. 155. A pattern.

An article on the use of the Marquoise Scales and Sector, page 126.

156 see (35). 157. To divide a triangle into any number of parts of equal area by straight lines parallel to the base. 158. To bisect a quadrilateral by a straight line through an angle. 159. To divide a triangle into three equal parts by straight lines from a given point in a side. 160. The Vernier Scale. 161. To draw a tangent to a given circle, or a flat curve nearly circular at a given point on it. 162. Problems on the inscribed and escribed circles. To construct a triangle having given the altitude, sum of sides and vertical angle, or the altitude, sum of sides and base.

Miscellaneous propositions without figures will be found on pages 100 *to* 105.

CHAPTER XVI.

SOLID GEOMETRY . . PAGE 137

163 to 176. Orthographic projection, and sections of solids.

EXAMPLES . . . PAGE 155

NOTES . . . PAGE 179

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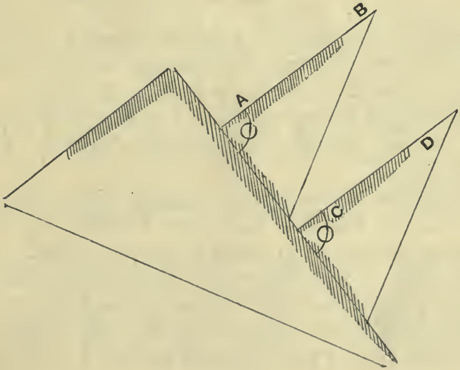
INTRODUCTION.

THE instruments necessary for a beginner are a flat ruler with scale shewing $\frac{1}{16}$ part of an inch, a compass, two pencils, set squares, and india-rubber. To these may be added a drawing-board, dividers, and T-square.

All construction lines should be drawn in clear, fine, continuous pencil lines with a moderately hard pencil, and figures, rather darker and thicker with an HH pencil. Faber's No. 4 hexagonal are suitable. Pencil points should be made chisel-shaped, but the points of compass pencils round.

Set squares are triangles of wood, or better of ebonite used for drawing parallel straight lines, the angles of these triangles are generally 30°, 60°, 90°, or 45°, 45°, 90°. Place one of these triangles against another with its edge AB in the position shewn in the figure. Keep the second triangle in the same position, but move the edge AB by sliding the first triangle along the

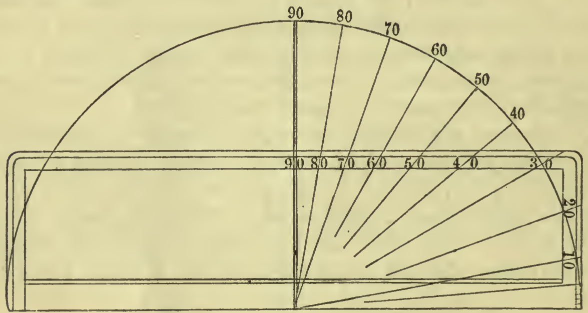


second till **AB** comes to **CD**. A straight line drawn along the edge **AB** will be parallel to one drawn along **CD**, and both at right angles to one drawn along the edge of the second triangle. [See also page 126.]

In simple problems it is better to make angles by construction [see Chapter II], but a protractor or flat bone ruler, six inches long, having angles marked on its edge, is generally used.

Angles may also be obtained more exactly by the use of the scale of chords [112].

The figure represents a protractor, and a semicircle with angles marked upon them.



In advanced examinations, especially those for Army Candidates, more instruments are required, and for really good work both compass and dividers should have *needle points.*

The additional instruments are a spring bow pen for the compass, as well as pencil; two good inking-in pens, and Marquoise Scales.

Measurements of lines should be taken from the protractor by means of the diagonal scale [see Chapter on Scales] by which lengths may be determined to $\frac{1}{100}$ part of half an inch.

When inking-in is to follow, it is best to use rather soft but well pointed pencils, and not to press too hard upon the paper, because the india-rubber removes the pencil lines more easily.

The pencils can be kept properly pointed by use of a file or sand-paper.

Liquid Indian ink is sometimes convenient, but solid Indian ink rubbed up fresh each time is best.

If, then, a candidate is going in for an examination where inking-in is required, he should be in the examination room in time to prepare the Indian ink and lake colouring before receiving his paper. Put a few drops of water into the Indian ink dish with a brush, rub in the Indian ink and keep adding drops as required. Prepare such a quantity and of such consistency that it may remain liquid to the end of the time, a drop or two of water being added and a little fresh Indian ink rubbed in from time to time.

Cover the dish to keep it from evaporating when not in use. Add a drop of oxgall to cause it to flow freely in the pens. Moisten the *inside* of drawing pens and fill with a brush.

When the drawing pen, being laid aside, does not start again properly, pass the point over a slightly moistened surface.

In using a pen compass, bend the leg of the compass so that the pen is at right angles to the paper, or nearly so.

Use a piece of chamois leather to clean the ink pens.

When dimensions of a figure are given or lengths of lines obtained, they are indicated by dotted lines and arrows [see figs. 153, 154].

Army Candidates should put letters and dimensions of figures in a red colour.

A few fine nibs should be kept in the drawing box for putting in letters and dimensions, and writing the calculations for scales.

In Geometrical Drawing no proof of propositions should be given, but only the number of the question, letters and dimensions indicated, and numerical calculations of scales, together with the name or index number of the candidate. Instructions on this point are usually given in the examination papers.

In Army examinations, the pattern and figure should be inked in. If time allow, construction lines can be inked in, either chain or dotted, but if not inked in, leave all construction lines in pencil. Dotted lines represent concealed lines in Solid

Geometry, and therefore chain lines are used by some draughtsmen in preference for construction. Otherwise fine dotted lines may be used for construction, and thicker ones for concealed lines. Army Candidates are advised to read the Text-books for Sandhurst and Woolwich in Military Topography and Fortification. It is not necessary to read these books beyond the chapter in scales in Military Topography, and Solid Geometry and Indices in the book on Fortification, and to look at further plates for style and colouring.

In drawing a straight line through two points, adjust the ruler so that the pencil point passes through both points when placed against the edge of the ruler, before drawing the line. When many lines pass through a point, begin to draw from this point. When a figure consists of curves and straight lines, ink in the curves before the straight lines. When inscribing a circle in a figure or circumscribing a figure by a circle, the centre required being found, make sure before describing it that the circle does pass through all the points required, by testing with the compass. If it does not do so, usually the defect can be remedied by *very slightly* altering the centre, or radius, or both before drawing the circle.

French curves are useful to construct conic sections and cycloids.

[For Elementary Science Course read specially (1) to (42), (47) to (109), (111) to (159), (163) to (176), and omit the article on Marquoise Scales and Sector: for Cambridge Local Examinations (1) to (114), (123) to (155).]

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PLANE GEOMETRICAL DRAWING.

CHAPTER I.

SIMPLE CONSTRUCTIONS.

A POINT has no parts and no magnitude but position. [Euclid, Defn. Bk. I.]

When a point is given its position is marked by a dot surrounded by a small circle.

A line is length without breadth and may be either straight or curved. A straight line lies evenly between its extreme points. Though the geometrical definition of a line states that it has no breadth, yet in representing a line in drawing it must have a certain breadth or thickness to make it visible; besides which by drawing lines thin, thick, dotted or chain certain sets may be distinguished from one another. For example lines required for construction are drawn thin, or dotted, but lines marking the solution of the problem rather thicker.

A superficies or surface has length and breadth but no thickness. A plane surface, or more shortly a plane, is such that any two points being taken in it the straight line between them lies wholly in that surface. The surface of a perfectly flat piece of paper is therefore a plane, and is called the plane of the paper. When we speak of Plane Geometry we mean the construction of figures which lie entirely in the plane of the paper, as distinguished from solid bodies which are represented by methods of projection.

When figures are contained by straight lines they are called rectilineal figures. A triangle is said to be a plane rectilineal figure because it is contained by three straight lines all lying in the plane of the paper.

A circle is a figure in a plane contained by one curved line such that all points in this line are equally distant from a point within it called the centre.

A circle then is a plane figure, but not a plane rectilineal figure. An angle is the inclination of two straight lines to one another. When a straight line meets another straight line so as to make the adjacent angles on opposite sides of it equal to one another, each of these angles is called a right angle, and the first straight line is said to be a perpendicular to the second.

In constructing the figures of Euclid's propositions straight lines are drawn of indefinite length and circles are completely constructed, when a very short arc would be sufficient. In Geometrical Drawing we do not produce lines or circular arcs further than is necessary.

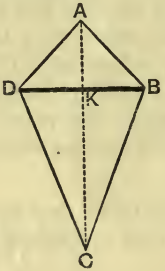
It is not necessary to give any proof or description of your work in Geometrical Drawing, nor is it necessary in the solution of problems to indicate points by letters unless this is done in the question.

When letters are necessary for the description of a figure they should be neatly printed, and words as in Italic printing.

The word "equals" is represented by =; feet by one dash, and inches by two dashes, thus 6′ 4″ means six feet, four inches.

The signs "plus" and "minus," + and − denote addition and subtraction.

1.



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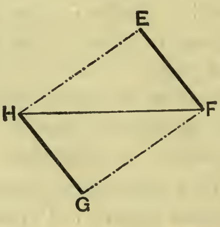

Take the two quadrilateral (or four-sided) rectilineal figures ABCD, EFGH. In the first of these let AB=AD and CB=CD, and in the second EH=FG and GH=EF. Let the diagonals AC, BD meet in K.

[As an illustration of the way of drawing straight lines BD, GH, and EF have been drawn thick; AC, dotted; EH and FG, chain.]

If the triangle ADC were laid upon ABC it would exactly coincide with it so that the angle DAC=BAC; also the triangle AKD, AKB are equal, DK=KB, and the angle AKD=AKB. This may be shewn to be the case by Euclid's method of superposition or laying one triangle upon another.

We also find the triangle EFH=GHF, so that the angle EFH=GHF. When a straight line HF meets two other straight lines GH and FE in this manner, EFH and GHF are called alternate angles, and if these angles are equal, however far GH and EF are produced, they will not meet in either direction and these straight lines GH and FE are said to be parallel.

From these figures we draw the following inferences.

If we want to bisect a given angle DAB *we must find a point* C *such that* CD=CB *and* AD=AB, *then join* AC.

If we want to bisect a given straight line BD *we must find a straight line passing through two points* A *and* C *such that* AD=AB *and* CD=CB.

If an angle is a right angle as AKB, *it must be such that if* BK *were produced to* D *making* KD=KB *then* AD=AB.

If a triangle as ABD *has two sides* AD *and* AB *equal the angles* ADB *and* ABD *are equal, and vice versâ.*

If a four-sided rectilineal figure has its opposite sides equal they are also parallel.

(1) To bisect a given angle DAB.

Mark off two equal distances AD, AB. With centres D and B and any convenient *equal* distances make arcs to intersect in C. Join AC. AC bisects the angle DAB.

(2) To bisect a given straight line BD.

With centres B and D and any convenient *equal* distances make arcs to intersect in A and C. The straight line AC bisects BD in K.

(3) To draw a perpendicular to DB from a given point K in it.

Take two points B and D such that KD = KB.

With centres D and B and at any convenient equal distances make arcs to intersect in A.

Join AK. AK is perpendicular to BD.

(4) From any given point A without a straight line BD to draw a perpendicular to it.

With any convenient distance and centre A make arcs to meet BD in B and D.

With B and D as centres and any equal distances make arcs to intersect in C. Join AC. AC is perpendicular to BD.

(5) To construct an equilateral triangle on a given base BD.

Draw BD of the length given in the question.

With centres B and D and distances equal to BD make arcs to intersect in A. ABD is an equilateral triangle.

An equilateral triangle is also equiangular, each of its angles being equal to one-third part of two right angles.

(6) To construct an isosceles triangle on a given base BD and with the other sides of given lengths.

[*An isosceles triangle has two equal sides, and therefore two equal angles.*]

Draw BD of the length given in the question.

With centres B and D make arcs at a distance equal to the given length of each of the other two sides. Let these arcs meet in A. Then ABD is the required triangle.

An acute angle is less than a right angle.

An obtuse angle is greater than a right angle.

An acute-angled triangle has three acute angles.

A right-angled triangle has a right angle.

An obtuse-angled triangle has an obtuse angle.

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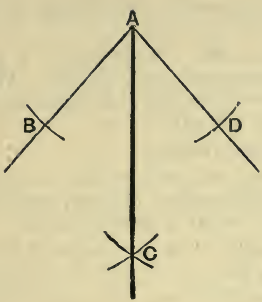

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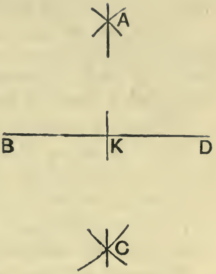

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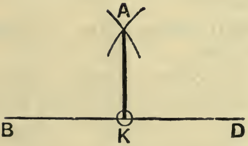

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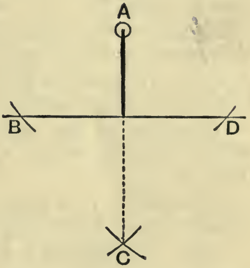

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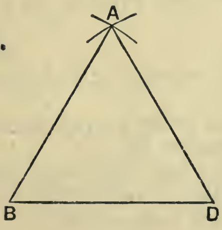

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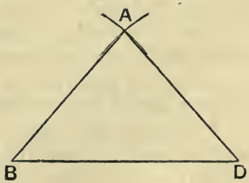


(7) To construct a scalene triangle ABC, the lengths of the sides of which are given.

Measure off and draw the base BC. With centre B and distance equal to one of the given sides describe an arc. With centre C and distance equal to the remaining side make an arc to intersect the first arc in A. Join AB, AC.

A scalene triangle has all its sides unequal.

We can draw four equal scalene triangles on the same base, first by reversing the order of the sides above the line, and then by drawing below the base triangles equal to those above it.

(8) To construct a triangle ABC when the angle ABC is given and the lengths of BA and BC.

Measure off the lengths BA and BC along the straight lines BA and BC. Join AC.

(9) To describe a square upon a given straight line AB.

At the point A draw a straight line AD perpendicular to AB and equal to it (see page 14, fig. 19). With centres B and D, and distances each equal to AB make arcs to intersect in C. Join BC and DC.

A square is a four sided rectilineal figure, having all its sides equal, and its angles right angles.

(10) To describe a rhombus ABCD having given a diagonal AC and the length of one of its sides AB.

Describe on the base AC two isosceles triangles (page 4) ABC, and ADC, one on one side and one on the other side of the base.

A rhombus is a quadrilateral rectilineal figure that has all its sides equal, but its angles are not right angles.

(11) To describe a rhombus ABCD when the angle ABC is given and one of its sides.

Draw the angle ABC included by the two straight lines AB, BC each of length equal to the given side.

With centres A and C and distances each equal to the given side make arcs intersecting in D.

Join AD, CD.

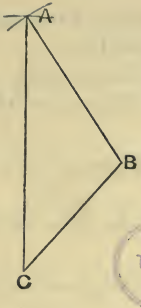
The diagonals of rhombus AC, BD *intersect at right angles at* K *and bisect one another* (fig. 17).

(12) To construct a parallelogram EFGH having given the lengths of the diagonal HF and the sides HE, EF.

Construct the two scalene triangles HEF and FGH above and below the base HF where FG = HE and GH = FE.

A parallelogram is sometimes called a rhomboid.

7.



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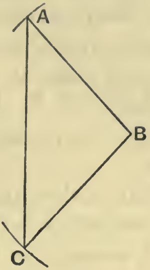

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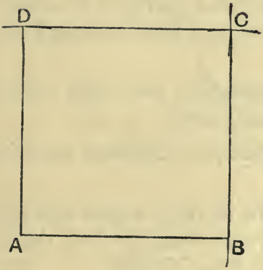

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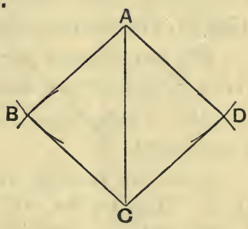

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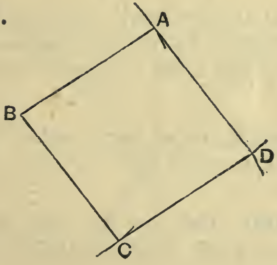

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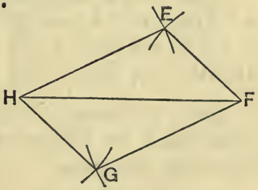

(13) To construct a parallelogram EFGH having given one angle EHG and the lengths of the sides HE, HG.

With centre E make an arc at distance = HG, and with centre G make an arc to intersect it at distance = HE to meet the first arc in F. Join EF, FG.

(14) To construct a rectangle EFGH having given the two sides.

Make a right angle EHG, measure off the sides HE and HG, and proceed as in the last example.

A rectangle or oblong is a right-angled parallelogram.

(15) Through a given point E to draw a parallel to a given straight line HG.

With centre E and any convenient distance make an arc to meet HG in H, with the same distance mark off HG, and still keeping the same distance mark off arcs from centres E and G. Join EF, the line required.

The construction for the rhombus has here been used, that for a parallelogram will do equally well.

(16) To describe a square ABCD, the diagonal AC being of given length.

Draw two straight lines AC, BD at right angles intersecting in K. Measure KA, KB, KC, KD each equal to half the given length of the diagonal. Join AB, BC, CD, DA.

(17) To describe a rhombus ABCD where the lengths of the diagonals AC, BD are known.

Draw two straight lines AC, BD at right angles meeting in K. Measure KA, and KC each equal to half of one diagonal, and KB, KD each equal to half of the other diagonal. Join AB, BC, CD, DA.

(18) To construct a rectilineal figure of any number of sides, such as ABCDE where the lengths of its sides and diagonals are given.

First construct the triangle ABC; then on the base AC describe the triangle ACD; next on AD describe ADE, and so on.

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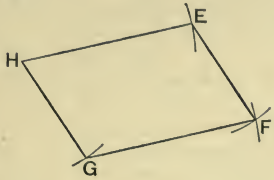

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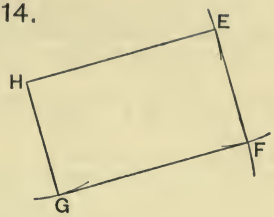

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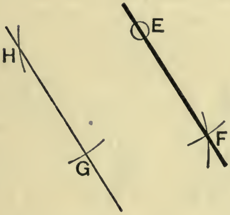

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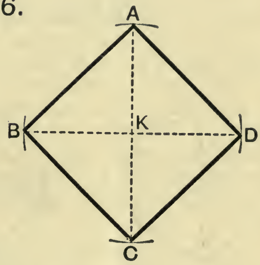

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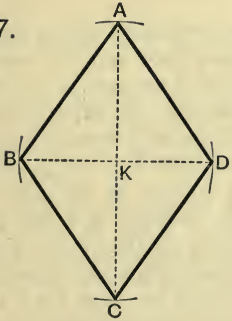

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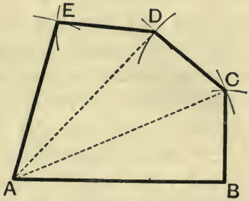

CHAPTER II.

MEASUREMENT OF ANGLES.

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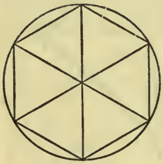

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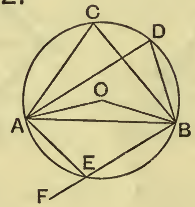

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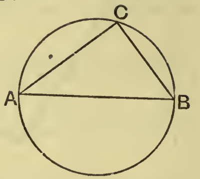

An angle has been defined as the inclination of two straight lines to one another. If we describe a circle and in it take two equal arcs and join the extremities of these arcs to the centre we obtain two equal angles at the centre. In any circle draw two diameters at right angles, and take one of the quadrants, or quarter circles. If we divide this quadrant into any number of equal parts and join the points of division to the centre we divide a right angle into the same number of equal parts. Since all right angles are equal to one another, each of these divisions is a definite fixed angle. Now when we measure any quantity we must take a fixed quantity of the same kind called a unit, and compare the quantity to be measured with the unit: thus if we know that one inch is a fixed length, then $3\frac{1}{2}$ inches means a length of three and a half times the given unit.

Suppose a right angle divided into 90 equal parts, by dividing the quadrant into 90 equal arcs. Each of the angles at the centre is $\frac{1}{90}$ part of a right angle and is therefore a definite fixed angle.

All angles are measured by comparing with this unit which is called a degree [1°]*.

It is not necessary to divide the quadrant into 90 equal parts and then take a certain number of them each time we find an angle: for instance to find an angle of 30° we do not divide the whole right angle into 90 equal parts and take 30 of them, but we remember that 30° is $\frac{1}{3}$ of 90°, the whole right angle, and take $\frac{1}{3}$ of a right angle by dividing the quadrant into three equal arcs.

Draw any circle (fig. 1), and step off the radius round the circumference you will find it will go round exactly six times. This does not mean that the circumference is six times the radius for each of the six arcs is greater than the radius which is its chord. What we do find however is this, that if we join each point of division to the centre, and also draw the six consecutive chords that each angle at the centre is $\frac{1}{6}$ of four right angles or 60°, that we have six equilateral triangles, and that the whole figure inscribed in the circle is a regular hexagon, that is a six-sided rectilineal figure the sides and angles of which are equal to one another.

The following rules should also be remembered.

Angles ACB, ADB in the same segment of a circle, centre O, are equal to one another and each half of the angle AOB (fig. 2).

The angle ACB in the semicircle ACB is a right angle (fig. 3).

If a quadrilateral rectilineal figure be described in a circle namely ACBE (fig. 2), then the sum of two of the opposite angles ACB, and AEB are together equal to two right angles: and if one of the sides BE be produced to F the angle AEF = ACB.

A segment of a circle is that part bounded by a chord as AB, and the arc which it cuts off as ACDB (fig. 2).

A sector of a circle as OAEB (fig. 2) is the part bounded by two radii as OA, OB and the arc AEB between them.

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* A degree is subdivided into 60 equal parts called minutes = 60′. A minute is subdivided into 60 equal parts called seconds = 60″. The measurement of angles by minutes and seconds is not used in this book.

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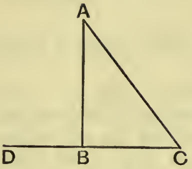

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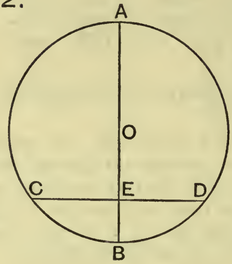

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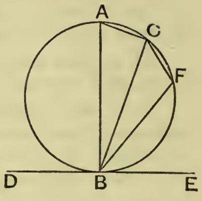

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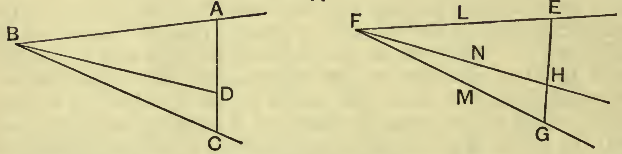

The exterior angle ABD of a triangle ABC (fig. 1) is equal to the sum of the interior angles BAC and BCA.

As a special case of the above take BA = BC so that the angles BAC and BCA are equal, then the angle ABD is double of BAC or BCA.

When in any circle a diameter bisects a chord it cuts it at right angles (fig. 2).

A straight line AB which bisects a chord CD at right angles passes through the centre, and also bisects the arc CBD in B.

To find the centre of any circle bisect any two arcs, which is the same as bisecting their chords, the bisecting lines meet at the centre.

Chords equally distant from the centre in a circle are equal to one another. The distance from the centre is measured by the perpendicular from the centre upon the chord.

A straight line DE (fig. 3) drawn through B at right angles to the diameter AB of a circle does not again meet the circle and is said to touch the circle. DE is called the tangent at B.

When a tangent DE and a chord BC are drawn through the same point B, the angles in the alternate segments are equal, that is the angle DBC = BFC and EBC = BAC.

The following construction (fig. 4) illustrates a property of similar figures.

Take two straight lines AB, BC intersecting at B and AC meeting them. Draw LE, MG, EG parallel to BA, BC, AC, and let AD be the same fraction of DC that EH is of HG. Then HN drawn parallel to DB will meet EL and GM in the same point F: and if DB bisects the angle ABC, HF will bisect the angle EFG.

This method is used for drawing straight lines to pass through points outside the drawing, for instance two straight lines being given do not intersect within the paper to draw a straight line that would bisect the angle between them if produced.

(19) Let AB be a given straight line it is required to draw from A a straight line perpendicular to it without producing AB.

With centre A make an arc BCFD rather less than a semicircle. Step off the arcs BC, CD with distance equal to the radius AB. Still keeping the same distance make arcs to intersect at E, with centres D and C. AE bisects the angle DAC and is perpendicular to AB.

(20) Let AC be a given straight line, and B a point nearly above A, it is required to draw a perpendicular to AC without producing it.

Take any point C in AC. Join BC and bisect it in O. Describe a semicircle cutting AC in A. Then the angle BAC is a right angle, and BA is perpendicular to AC.

(21) To construct an angle 60° at a point A, on the straight line AB.

Describe an arc BC with centre A and any convenient distance. With centre B and the same distance mark off a distance BC. The angle CAB = 60°.

(22) To construct an angle 30° at a point A, on the straight line AB.

First make an angle CAB = 60°, with centres C and B still keeping the same distance = AB describe arcs to intersect in D. Then DA bisects the angle CAB therefore DAB = 30°.

(23) To construct an angle 15° at a point A on the straight line AB.

Bisect the angle 30° by the straight line AK, then KAB = 15°.

(24) To construct an angle 45° at a point A on the straight line AB.

In the figure (19) for finding a right angle make arcs with centres F, and B still with distance = AB to meet at G. AG bisects the right angle FAB, therefore GAB = 45°.

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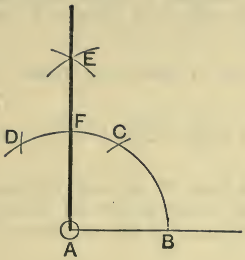

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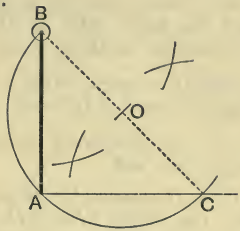

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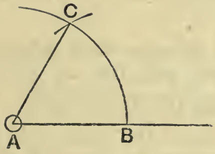

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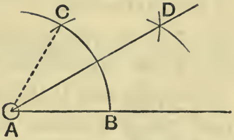

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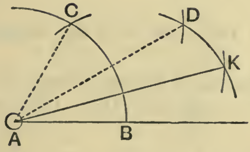

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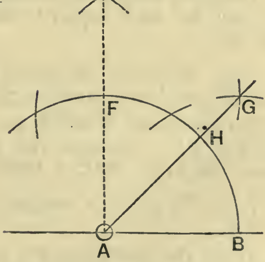

(25) To construct an angle of 75° at the point A on the straight line AB.

Still taking the figure (19) for finding a right angle at A, bisect the angle CAF by describing arcs with centres C and F and distance = AB to meet in M.

Then the angle MAB = the sum of CAB and CAM = 75°, being the sum of 60° and 15°.

(26) To construct an angle of 105° at the point A, on the straight line AB.

With the same figure as in the case of 45° and 75° (24, 25) bisect the angle FAD by a similar construction to that used for 75°. Then the angle NAB = the sum of the angles 90° and 15° = 105°.

In the same way by taking arcs subtending 15° *at the centre and adding them on or taking them from the arcs subtending angles already known we can find any angles from* 0° *to* 180°, *differing by* 15°, *at the centre of the circle.*

Other angles are found by dividing arcs by trial, or by use of the protractor.

(27) To describe a right-angled triangle CAB where the lengths of the hypotenuse AB and one side CB are given.

[The hypotenuse is the side opposite the right angle.]

Draw AB and bisect it at O. Make the semicircle ACB. Mark off the given distance CB with centre B. Join CA, CB.

(28) To describe a rectangle ACBD where the lengths of the diagonal and one side are given.

Draw the diagonal AB and bisect it in O.

Describe the circle ACBD with centre O. Mark off distances BC and AD each equal to the other given side. Join AC, CB, BD, DA.

(29) To make an angle BAC at the point A in the straight line AC equal to a given angle DEF.

With centres A and E and any the same distance make arcs BC and DF. Measure the arc BC = the arc DF. Join AB. Then the angle BAC = DEF.

(30) To draw a tangent to a given circle at a given point B on it.

Draw the diameter AB, and CD at right angles to it at B. CD is the tangent required.

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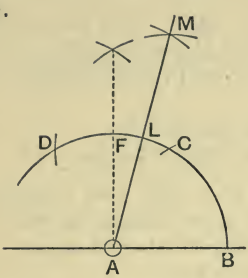

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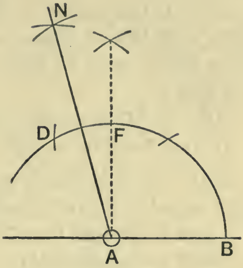

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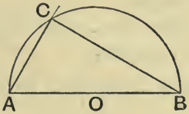

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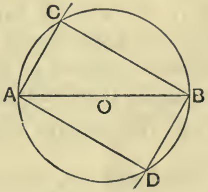

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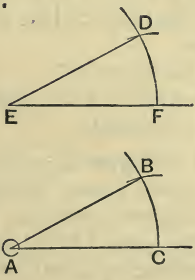

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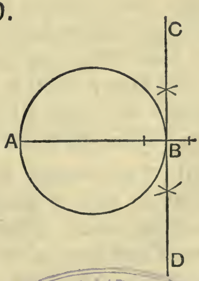

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(31) To draw tangents to a given circle from a given external point A.

Find O the centre of the given circle. Join OA and draw a circle with OA as diameter [that is bisect OA at M and draw a circle with centre M and distance MA] cutting the given circle at B and C.

AB and AC are tangents to the given circle.

[If one tangent only is required draw AB or AC.]

(32) To describe a segment of a circle AEB to contain an angle equal to a given angle, on a given straight line AB.

Draw a straight line CD bisecting AB at right angles.

Make the angle BAQ = the given angle.

Draw AO perpendicular to AQ meeting CD in O. A segment described with centre O and distance equal to OA contains an angle equal to BAQ.

(33) To cut off from a given circle a segment ABC to contain an angle equal to a given angle.

Draw a tangent at a point A. Make the angle DAC = the given angle, then ABC is the segment required.

(34) To find the centre of a circle of which a part ABC is given.

Bisect any two arcs AB, BC which form part of the given segment. The two bisectors meet at the centre.

[An arc is bisected exactly in the same way as a straight line.]

(35) Two straight lines DE, CF are given in position, it is required to find without producing them the straight line which would bisect the angle between them.

Draw from a point A in DE, AG parallel to CF.

Bisect the angle DAG by AB, and bisect AB at right angles by HK. Then HK is the straight line required.

(36) Two straight lines DE, CF are given in position, it is required to find a straight line which passes through their intersection, and a given point P, without producing them.

Draw any two parallels LM, AB. Join AP, BP and draw LQ, MQ parallel to AP and BP intersecting in Q. PQ is the straight line required.

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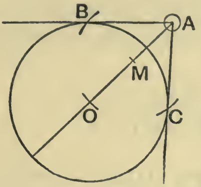

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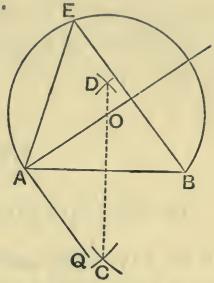

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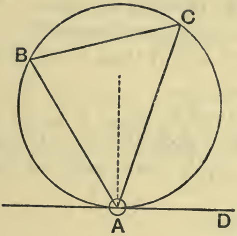

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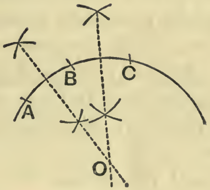

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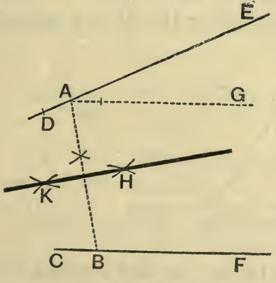

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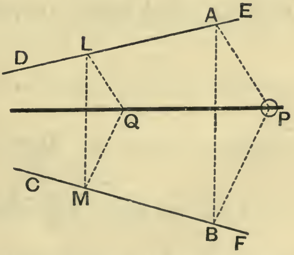

CHAPTER III.

ON PROPORTION OF STRAIGHT LINES.

If we have four numbers represented by a, b, c, d and a is the same fraction of b that c is of d then a, b, c, d are said to be proportionals.

The relation of a to b, or c to d is called a ratio.

Proportion is sometimes defined as equality of ratio.

a and c are called antecedents; b and d, consequents.

For example suppose $a = 4$ and $b = 6$, if then $c = 8$ we find $d = 12$ for 4 is $\frac{2}{3}$ of 6 and 8 is $\frac{2}{3}$ of 12.

a is the same fraction of b that c is of d.

If there are three quantities a, b, c such that a is the same fraction of b that b is of c, then a, b, c are said to be in continued proportion.

b is a mean proportional to a, and c.

c is a third proportional to a and b.

Euclid's propositions about proportion of lines are deduced from the fact that if two triangles are equiangular the sides about the equal angles are proportional, so that if two straight lines inclined at any angle are cut by two parallel straight lines they are cut proportionally for (figure 1) if the two straight lines

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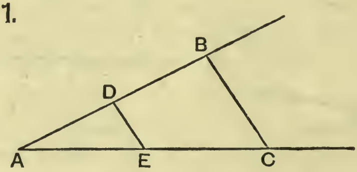

AB, AC are cut by the parallels DE, BC the triangles ADE, ABC are equiangular.

If a perpendicular FH is drawn from the right angle GFK of a right-angled triangle GFK to the base, the triangles FGH, KHF are similar to the whole triangle GFK and to each other (figure 2).

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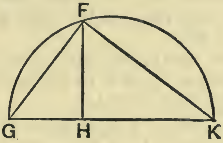

By the expression AD : AB we mean the ratio of the length of AD to that of AB, AD : AB : : AE : AC or AD : AB = AE : AC means that the fraction AD is of AB is the same that AE is of AC.

The following equalities of ratio should be learned and understood from the above figures, remembering that the sides of equiangular triangles about equal angles are proportional.

AD : AB : : AE : AC.

AD : DB : : AE : EC.

AD : AE : : AB : AC.

AD : DE : : AB : BC.

The second of these ratios is derived from the first, for if $a : b :: c : d$ then $a : a - b :: c : c - d$.

GH : HF : : HF : HK so that FH is the mean proportional between GH and HK.

GK : GF : : GF : GH so that GF is the mean proportional between GK and GH.

Similarly FK is a mean proportional between KH and KG.

(37) To divide a straight line into any number of equal parts: for example one inch into five equal parts.

Draw AB one inch long, and any straight line AC making a convenient angle with AB.

Mark off any five equal divisions on AC.

Join the last of these points of division C to B, the parallels drawn to BC through the other points of division will divide AB into five equal parts.

(38) To divide a straight line into parts in any given proportion. For example divide one inch into three parts in proportion of 3, 4, 5.

Draw AB one inch long, and any straight line AC making a convenient angle with AB.

Mark off any 12 equal divisions on AC for $3+4+5=12$. Take 3, then 4, then 5 of these. Join B to the last division, and draw parallels through the other points of division.

(39) To find a fourth proportional to three given straight lines, when the second is greater than the first. For example 4, 8, and 5.

Draw AB, AC at any angle. Measure $AD=4$, $AB=8$, $AE=5$, in this case take $\frac{1}{8}$ part of an inch as unit. Join DE and through B draw BC parallel to DE. AC will be found to measure 10, the fourth proportional. *The second parallel* BC *is beyond the first* DE.

The same method is used to find the third proportional to two given lines. For example if $AD=4$ and $AB=5$ and $AE=5$ making the same construction we find $AC=6\frac{1}{4}$, that is to say $6\frac{1}{4}$ is the third proportional to 4 and 5.

(40) To find a fourth proportional to three given straight lines, when the second is less than the first. For example 8, 4, 10.

Draw AB, AC at any angle. Measure $AB=8$, $AD=4$, $AC=10$. Join BC and through D draw DE parallel to BC. AE will be found to measure 5, the fourth proportional. *The second parallel* DE *is nearer to* A *than* BC.

The same method is used to find the third proportional to two given lines. As an example take the numbers 3 and 2. We find $3 : 2 :: 2 : 1\frac{1}{3}$.

(41) A straight line PS may be divided proportionally to another LO by a second method.

Place PS and LO parallel to one another.

Join PL and OS to meet in A.

If we join A to M and N the points of division of LO to meet PS in Q and R, PS will be divided at Q and R similarly to LO.

(42) To find a mean proportional to given straight lines. For example take $GH=4$ and $HK=9$.

Place GH and HK in the same straight line, and on GK draw the semicircle GFK, then HF perpendicular to GK is the mean proportional required $= 6$, for $4 : 6 :: 6 : 9$. [unit $=\frac{1}{12}''$.]

If we had taken the whole $GK=9$ and $GH=4$ then we find $GF=6$, the mean proportional. When the numbers are too large to make a convenient figure use this second method.

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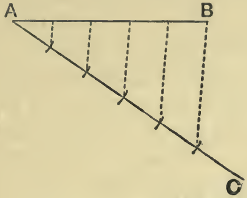

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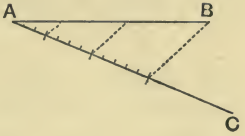

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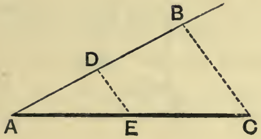

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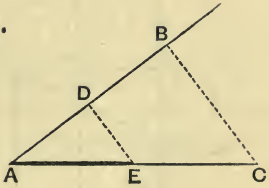

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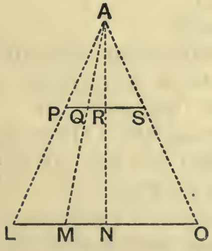

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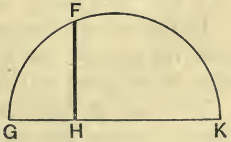

CHAPTER IV.

EXTREME AND MEAN RATIO.

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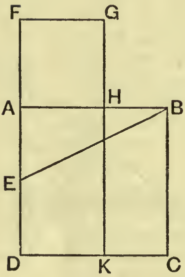

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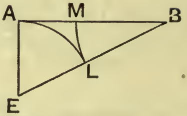

To divide a straight line AB in the point H so that the rectangle contained by AB . BH = square on AH, or in other words AB : AH :: AH : BH.

The construction given in Euclid II. 11 (fig. 1) may be shortened by that given above (fig. 2).

The triangle AEB is the same in each.

In Euclid's construction we take the straight line AB, on it describe a square ABCD; bisect AD in E; join EB, make EF = EB; and lastly on AF describe a square AFGH.

In the second construction we obtain BM = AH as follows. Draw AE at right angles to AB = $\frac{1}{2}$ AB; join BE, with centre E make arc AL, and with centre B make arc ML.

The use of extreme and mean ratio is chiefly to find a triangle having the angles at the base each double the third angle; for if a triangle be described on a base = BM and each of its equal sides = AB it is such a triangle, its angles are 72°, 72°, and 36°.

If we bisect the angle 36° we get 18°.

Again, a regular pentagon, or five sided rectilineal figure having all its sides and angles equal is such that if BM = one of its sides, then AB is one of its diagonals, that is a straight line joining alternate angles.

In any regular pentagon this ratio is the same, namely, side : diagonal :: BM : AB. Expressed numerically this ratio is $2 : \sqrt{5} + 1$. In a right-angled triangle the hypotenuse of which is $\sqrt{5} + 1$ and one side 1, the least angle is 18°.

These values may be tested for if we take $AB = \sqrt{5} + 1$, and $BM = 2$, then $AM = \sqrt{5} - 1$ so that $AH = 2$, and $BH = \sqrt{5} - 1$.

Multiply AB by BH we get 4 = the square on AH.

It is not necessary to introduce $\sqrt{5}$ into the rules to be remembered, except as an aid to the memory. It is sufficient to know if AB be divided in extreme and mean ratio in H, that a right-angled triangle hypotenuse = AB and one side = $\frac{1}{2}$ AH has its least angle 18°, and that the side and diagonal of a regular pentagon are in the ratio AH : AB.

(43) To divide the straight line AB in extreme and mean ratio.

Draw AE at right angles to AB = $\frac{1}{2}$ AB.

Join BE, with centre E make arc AL and with centre B make arc LM, then AB : BM : : BM : AM.

(44) To construct a triangle with angles 72°, 72°, and 36°.

Make an isosceles triangle with two of its sides = AB of the last proposition and its base = BM.

Another method is to find the position of the angle V (see fig. 46) and join AV, VB.

In the first case one of the equal sides is given, and in the second case the base is given.

(45) To inscribe a regular pentagon in a given circle.

Draw a diameter AB, and OF from the centre O, perpendicular to it.

Bisect OB at E. With centre E make an arc FH with distance EF, then FH is the length of the side of a regular pentagon in the circle. Measure it, and step it off round the circumference, joining the points BK, KN, NM, ML, LB.

The proof of this is somewhat complicated but it can be shewn by taking BE as unit, that EF = $\sqrt{5}$ (Euclid I. 47) from which by repeated use of the same proposition we find values for BK and BN that are in proportion of $2 : \sqrt{5} + 1$.

(46) To describe a regular pentagon upon a given base AB.

Bisect AB at right angles by VH. Measure DH = AB.

Produce AD to E so that DE = AH. With centre A describe an arc VE to cut HV in V.

V is the vertex of the pentagon. With centres V, A, and B and distance equal to AB make arcs to meet in F, and C. Join BC, CV, VF, FA.

If AB = 2. AH = 1. AD = $\sqrt{5}$, so that AV = $\sqrt{5} + 1$, therefore the angle AVH = 18°. VAB = VBA = 72°.

AVB = 36°.

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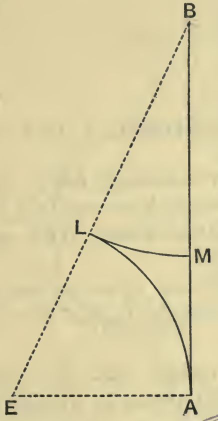

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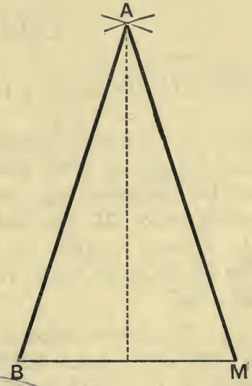

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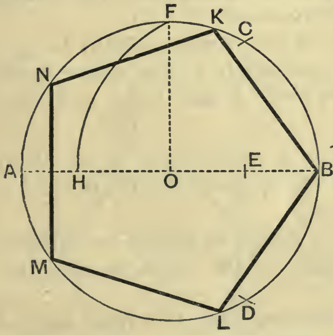

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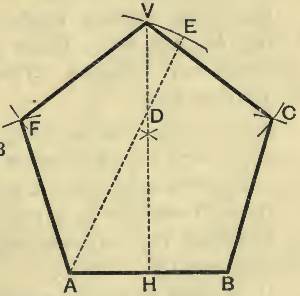

CHAPTER V.

INSCRIBED AND CIRCUMSCRIBING CIRCLES.

(47) To inscribe a circle in a given triangle ABC. Bisect two of its angles BAC, ABC by straight lines meeting at O. Draw OD perpendicular to one of the sides. With centre O and distance OD describe a circle.

To inscribe a circle in any rectilineal figure in which it is possible to do so, bisect two of its angles to find the centre of the circle.

(48) To circumscribe a given triangle ABC by a circle. Bisect two of its sides AC, CB at right angles by straight lines meeting in O. With centre O and distance OA = OB = OC describe a circle.

To describe a circle about any rectilineal figure, about which it is possible to do so, bisect two of its sides at right angles to find the centre of the circle.

(49) To describe a circle to touch the side BC of the triangle ABC and the sides AB and AC produced.

Bisect the exterior angles of the triangle formed by producing AB and AC. These intersect in O, the centre of the circle which is required. This circle is said to be *escribed* to the triangle ABC.

(50) To inscribe a circle in a quadrilateral figure ABCD which is such that the sum of the sides AD and BC is equal to the sum of the sides AB and CD.

If this condition be given, a circle can always be inscribed by bisecting two adjacent angles of the figure, as this fixes the centre of the circle.

It is usual to give a quadrilateral such that AB = AD and BC = CD as this ensures the condition that AB and CD together = AD and BC.

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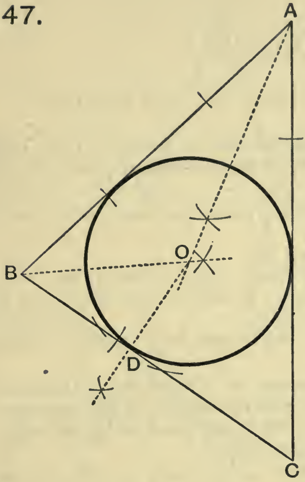

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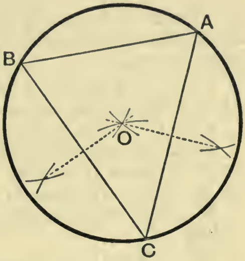

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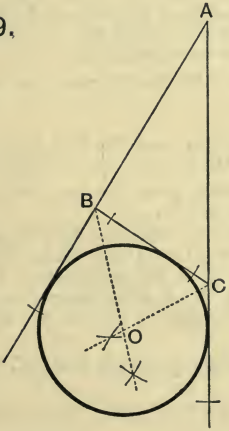

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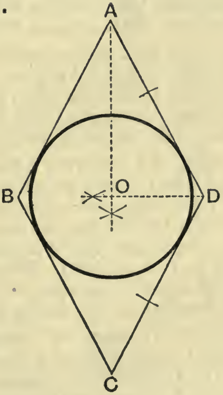

CHAPTER VI.

ON THE CONSTRUCTION OF TRIANGLES.

[SOME of the more simple constructions of triangles have been already given, and some depending upon more advanced principles will be given later.]

(51) To construct a triangle having given the length of the sides AC and BC and the angle CAB.

Case I. When AC is greater than BC.

Make the angle CAB equal to the given angle. Measure AC of the required length along CA. With centre C and distance BC make an arc BD which will meet AD in B and D.

Either of the triangles ABC or ABD will be a solution of the problem.

(52) Case II. of the above proposition. When AC is less than BC. In this case the arc described with centre C and distance equal to the known value of BC only meets AB in one point within the angle, and there is but one solution, the triangle ABC.

(53) To construct a triangle ABC having given the altitude AM and the angles at the base.

Draw the altitude AM, and DAE at right angles to it. With centre A make the semicircle DKLE, with any convenient radius. Make the angles KAD and LAE equal to the given angles at the base, if ABC be equilateral the distances KD and LE are each equal to AD. Draw BC parallel to DAE to meet AK and AL produced in B and C.

(54) To construct an isosceles triangle ABC having given the base BC and the vertical angle.

Make DBE = the given vertical angle.

Bisect the angle CBD by BA and draw to meet the line bisecting BC at right angles in the point A. Join AC.

(55) To construct a triangle ABC having given the base BC and the angles in given proportion, in this case as 1 : 2 : 3.

With any convenient radius describe the semicircle EFDH, and divide it by trial into six equal arcs because $1+2+3=6$. Take F and D as marking 2 and 3 of these divisions. Join BD and produce it to meet in A a parallel to BF drawn through C. It is necessary to state which angle is opposite to the given base, in this case the greatest angle has been taken opposite to the given base.

(56) To construct a triangle ABC having given the base AC, the vertical angle ABC, and the side AB.

On AC describe a segment to contain an angle equal to the given angle ABC (page 18, fig. 32).

Mark off AB with centre A and at the given distance. Join BC.

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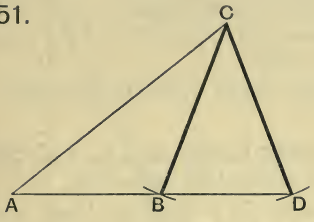

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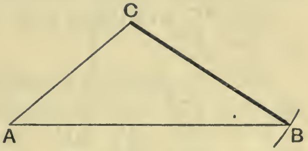

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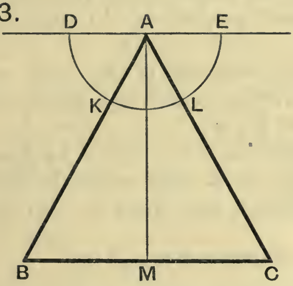

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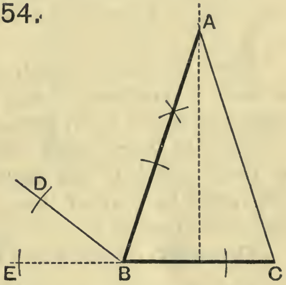

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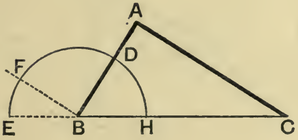

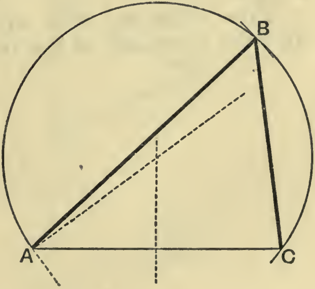

(57) To construct a triangle ABC having given the base AB, the angle BAC, and the sum of the other two sides.

Make the angle BAC, measure the length of AB; take AD = the sum of the other two sides.

Join BD and bisect it at right angles by CE to meet AD in C. Join BC.

(58) To construct a triangle ABC having given the base BC the angle BCA and the difference of the sides AB and AC.

Make the angle BCA. Measure CB = the given base, and CD the given difference of sides. Join BD, and bisect it at right angles by AE meeting CA in A. Join AB.

(59) To construct a triangle ABC having given the sum of its sides, and its angles in given proportion. In this case 2 : 3 : 4.

Draw DE of the length of the given sum of the sides. At D make a semicircle of any convenient radius, and divide the semicircle into 9 equal parts, $2 + 3 + 4 = 9$.

Take an angle ADM subtended at the centre D by three of these divisions and bisect it. At E take an angle BEN subtended by four of these divisions, and bisect it. Let the bisectors meet at C.

Through C draw CA and CB parallel to DM and EN.

(60) To construct a triangle ABC having given the sum of its sides and two of the angles at the base.

The construction is similar to that given in the last proposition.

Take ADM and BEN equal to the given angles, where DE = the given sum of the sides, and proceed as before.

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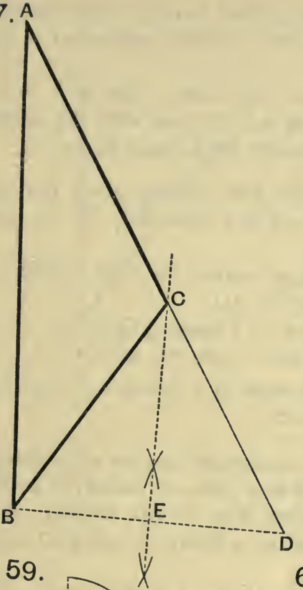

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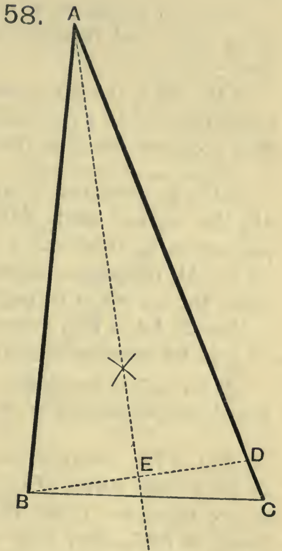

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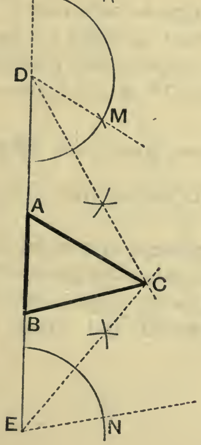

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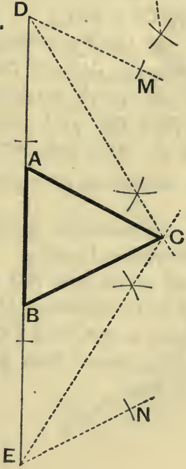

(61) To construct a triangle ABC having given the sum of its sides, and that they are in a certain proportion in this case 3, 4, 5.

Take DE = the given sum of the sides. Divide it in the proportion of 3 : 4 : 5 (page 22, fig. 38) and with the lengths thus obtained describe the triangle ABC (page 6, fig. 7).

(62) To construct a triangle ABC having given the base AB, the vertical angle ACB, and the sides AC, CB in given proportion, in this case 2 : 3.

On AB describe a segment to contain an angle = ACB, and bisect the arc AB at D (page 18, fig. 32).

Divide AB at E in proportion 2 : 3 (page 6, fig. 7).

Join DE meeting the circle in C. Join AC and CB.

If a straight line bisects the angle of a triangle it divides the base in the proportion of the sides.

(63) [*The straight lines joining the angles of a triangle to the middle points of the opposite sides are called the medians of the triangle. If we represent these by* AD, BE, CF *in the triangle* ABC, *then they meet at a point* O, *and* $OD = \frac{1}{3}AD$, $OE = \frac{1}{3}BE$, $OF = \frac{1}{3}CF$.]

Given the three medians of a triangle to construct it.

Describe the triangle OCK such that its sides are each two-thirds of the given medians, and complete the parallelogram COBK. Join BC, and produce OA = OK in the same straight line with it. Join AB, AC.

[*The diagonals of a parallelogram bisect each other.*]

(64) To construct a triangle ABD having given the altitude, base, and vertical angle.

On the given base AB draw a segment to contain the given angle (as in 62 above). Draw AC equal to the given altitude at right angles to AB, and CD parallel to AB meeting the segment in D. We obtain two positions for D, and either will do. Join AD, BD.

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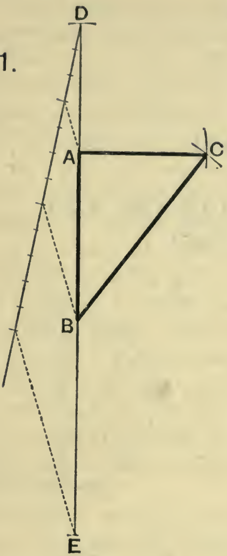

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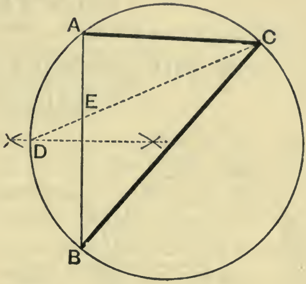

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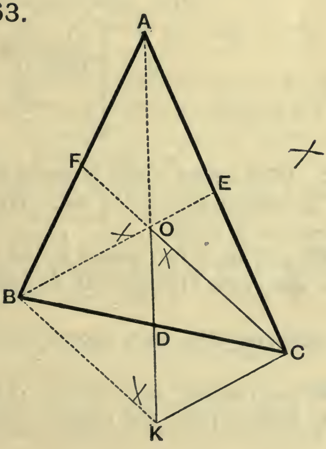

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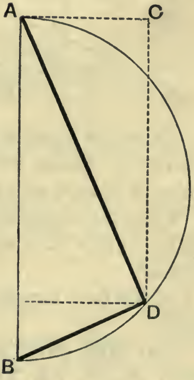

CHAPTER VII.

ON THE CONTACT OF CIRCLES AND STRAIGHT LINES.

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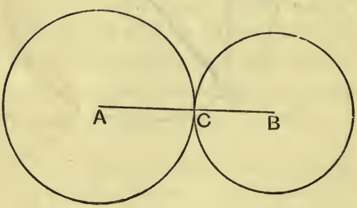

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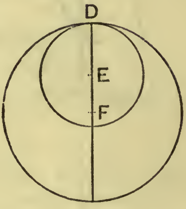

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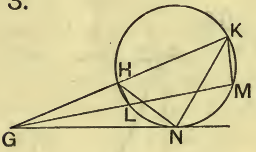

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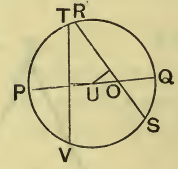


Take the straight line AC. With centre A and distance AC describe a circle, where AC is any given part of AB. With centre B and distance BC describe a circle.

Take the straight line DEF, and in it two points E and F. With centres E and F and distances ED and FD describe circles.

Let GN be a tangent to a circle and GHK, GLM chords of the circle.

Let ROS, POQ be chords intersecting in a circle. Two equal chords RS, TV equally distant from the centre U are equal.

The distance of a point from a straight line is measured by the perpendicular from the point to the line.

When two circles (fig. 1) touch one another the distance between their centres is the sum of their radii for external contact, or the difference of their radii (fig. 2) when one circle touches another internally.

If we join the lines HL and KM so as to form a quadrilateral in the circle the exterior angles GHL and GLH are respectively equal to the angles GMK and GKM (see page 12) so that the triangles GHL and GMK are equiangular.

Therefore using the rule about the proportion of the sides of equiangular triangles (page 20) we find GH : GM :: GL : GK.

If the triangle GHL were turned over so that GL lies along GK and GH along GM, then HL would become parallel to KM.

By taking the rule of the equality of angles in the alternate segment (page 12) we find the angles GNH, HKN are equal, and the triangles GHN, GNK are equiangular so that GN is the mean proportional between GH and GK by using the similar triangles GHN and GNK.

Triangles are said to be similar when they are equiangular for, as we know, the sides about the equal angles are proportional (page 20).

In the same way GN is the mean proportional between GL and GM.

In the circle RQSP angles in the same segment are equal so that the angle RPQ = RSQ and the angle PRS = PQS.

Therefore the triangles ORP and OQS are similar, therefore OR : OP :: OQ : OS.

In the propositions which follow a note is sometimes made that there are two solutions.

When a circle is drawn to meet another circle or straight line it usually does so in two points, and when one of these intersections gives a solution of a problem the other one does so also.

For example take number 67 page 39, the circles centres A and D distances AC and DF intersect in another point, on the other side of AD which is also a centre of a circle of given radius touching both given circles.

(65) To draw two tangents, externally, to two given circles.

Let AC be the radius of the greater of the two given circles, and B the centre of the smaller.

Measure CD = radius of the smaller given circle, so that AD is *the difference of the radii of the given circles.* Proceed as if we were drawing tangents from B to the circle of radius AD (fig. 31), this determines the points K and L. Join AK, AL to cut the greater of the given circles at E and G. Straight lines at right angles to AE and AG are the tangents required.

(66) To draw two tangents, internally to two given circles.

Proceed as in the last proposition making the following change, measure CD outside the greater circle so that AD is *the sum of the radii of the given circles.*

[The construction of both of these figures depends upon the fact that KG = CD, and that KGHB, and LBFE are rectangles.]

(67) To draw a circle of given radius to touch two given circles externally.

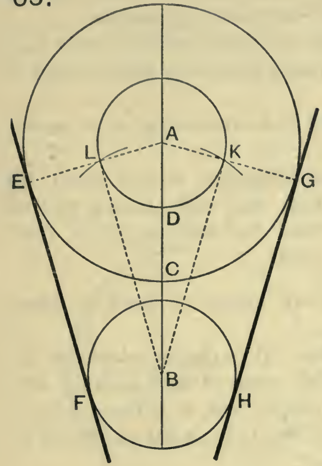
Let AB and DE be the radii of the given circles. *Add the given radius to each* by measuring BC and EF equal to the given radius, and with centres A and D and radii, respectively, AC and DF make arcs to meet in G. G is the centre of the required circle.

(68) To draw a circle of given radius to be touched internally by two given circles.

Let AE and BD be the radii of the given circles. *Subtract each from the given radius* by measuring EF and CD each equal to the given radius, and with centres A and B and radii, respectively, AF and BC make arcs to intersect in G. G is the centre of the required circle.

[*There are two solutions to* 67 *and* 68.]

65.



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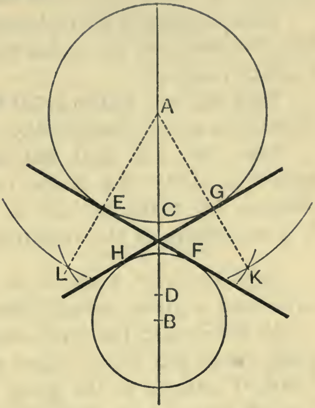

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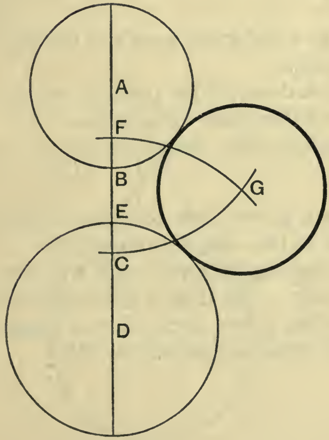

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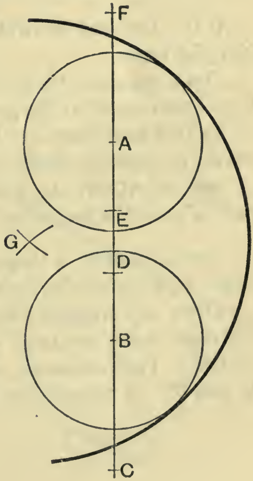

(69) To draw a circle of given radius to be touched externally by one given circle and internally by another.

This proposition depends upon the same principle as 67 and 68. We must *add the radius* to touch *externally* and *subtract it* to touch *internally.*

Take AC, and add to it CD the given radius and with centre A make an arc at distance AD.

Take BH and find BD the difference of the radii by measuring DH = the given radius, and with centre B and distance BD make an arc to meet the first arc in F, then F is the centre of the required circle.

(70) To draw a circle of given radius to touch a given circle and a given straight line.

As before add the given radius CD to the radius AD of the given circle and make an arc with centre A and distance AC. Draw BF parallel to the given straight line, at a distance from it equal to the radius (which is given) to meet the arc BC in B. B is the centre of the circle required.

[*There are two solutions to* 69 *and* 70.]

(71) To draw a circle of given radius to touch two given straight lines.

Draw parallel straight lines to the given ones at a distance from them equal to the given radius.

In this case there are four solutions, for the parallels may be drawn to meet on either side of either given straight line.

AB and AC are the given straight lines, and in the figure one pair of parallels meets at O.

(72) To draw a tangent to a given circle to make a given angle with a given straight line, in this case the tangent BD.

Draw any straight line DM making the given angle with BD. Through A the centre of the given circle draw a perpendicular to DM. This diameter meets the given circle in two points N and P. A tangent at either point is parallel to DM.

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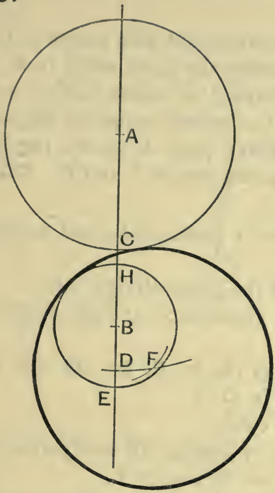

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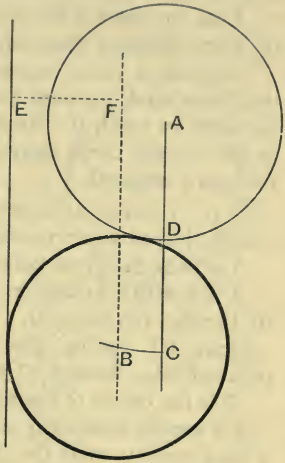

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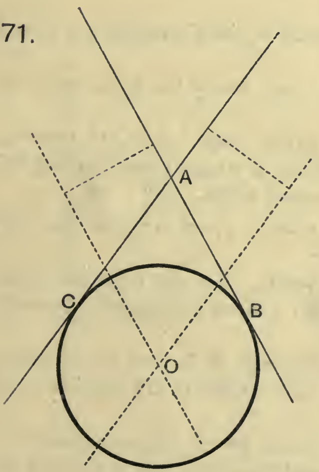

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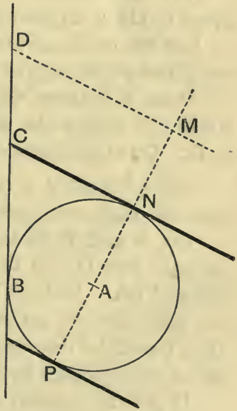

(73) To draw a chord of given length in a given circle to pass through a given point.

Take any point A on the given circle, and with centre A and the given distance make an arc to meet the given circle in B.

Describe a circle concentric (having the same centre) with the given circle to pass through the middle point of AB, and therefore to touch it. From the given point O draw a tangent to this second circle cutting the given one in C and D. CD is the chord required.

(74) To draw a circle to touch a given circle, and a given straight line at a given point.

Let O be the given point in OF the given straight line.

Draw ADBF a diameter of the given circle at right angles to OF through the centre D.

Join AO cutting the given circle in E, let DE cut the perpendicular through O to OF, in C.

C is the centre of the required circle.

[A second solution is obtained by joining OB and producing it backwards to meet the given circle in a point K.

KD will meet OC in a point M, a circle centre M and radius MK will give another solution.]

(75) To draw a circle to touch a given straight line and a given circle at a given point.

Let BE be the given straight line, and O the given point on the given circle, centre A.

Draw a tangent OB to the given circle. Join OA meeting BE in C. Bisect the angle OBC by a straight line meeting OC in D. D is the centre of the required circle.

(76) To draw a circle to touch a given straight line and pass through two given points.

Let A and B be the given points. Let AB meet the given straight line DT in D. Find DC a mean proportional between DB and DA. Make DT = DC.

The required circle passes through A, B, and T, its centre O is found by drawing OT at right angles to DT and bisecting BA at right angles by OH.

If AB is parallel to DT, bisect BA at right angles by a straight line meeting DT in T, and describe a circle through the three points A, B, and T (fig. 48).

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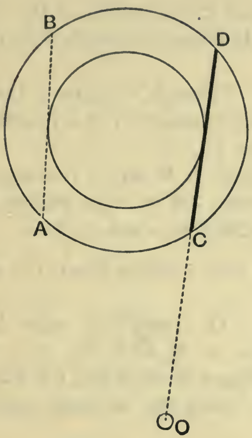

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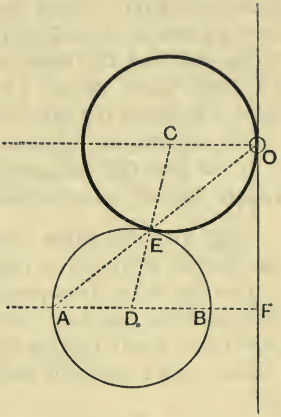

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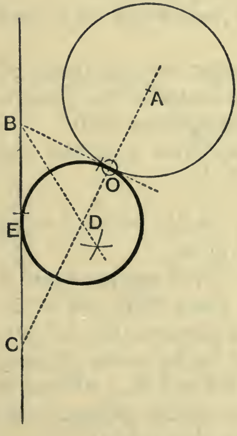

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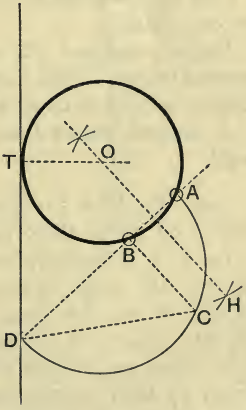

(77) To draw a circle through two given points to touch a given circle.

Let A, B, be the given points and C the centre of the given circle, radius CT. Draw any circle passing through A and B to cut the given circle in D and E.

Let AB and DE meet in O. Through O draw a tangent OT to the given circle. [Another solution to the problem is found by taking the other tangent.]

The required circle passes through A, B, and T (see fig. 76).

If AB and DE are parallel, bisect AB at right angles by a straight line HE, which will meet the given circle in T.

(78) To draw three circular arcs each to touch the other two, having three given centres.

Join the three given points A, B, C. Inscribe a circle in the triangle ABC, touching the sides in D, E, F (fig. 47). The required arcs with centres A, B, C pass through DE, EF, FD.

For circles touching internally, take the escribed circles of ABC.

(79) On a given straight line to find a point from which we can draw tangents of given length to a given circle.

[*All tangents drawn to a circle from points on a concentric one are equal.*]

Draw a tangent CE at any point on the circle of the required length. With centre A, the centre of the given circle, make an arc DE to meet the given straight line in D.

A tangent drawn from D to the circle is of the required length.

(80) To draw a segment of a circle, (without using the centre,) on which three points are given.

Let A, B, C be the given points. With centres A and B and distance AB, make circular arcs. Draw BCD and ACE. On the arcs AK and BH mark off equal distances DL, LM, DK, EF, EG, GH. Since, in a circle, angles in the same segment are equal, when we join AH and BM; AG and BL; AF and BK we obtain by their intersections points on the segment which is drawn, freehand, through the points thus found.

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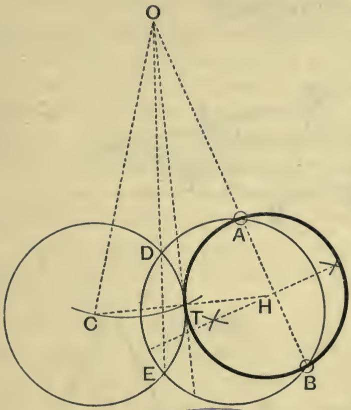

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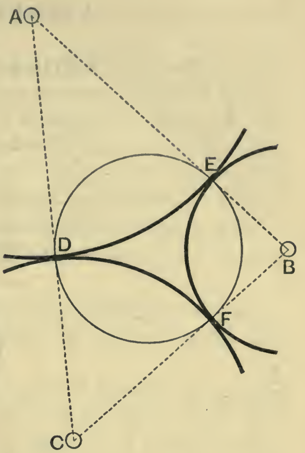

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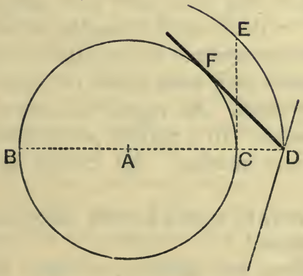

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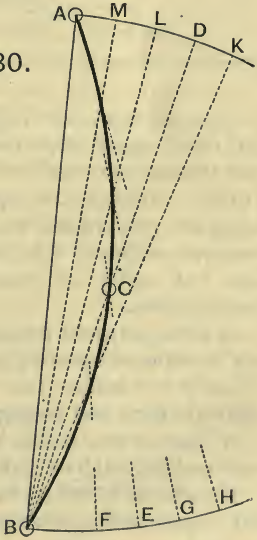
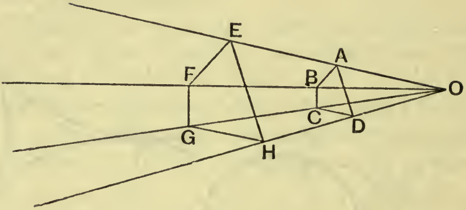


CHAPTER VIII.

SIMILAR FIGURES.

1.



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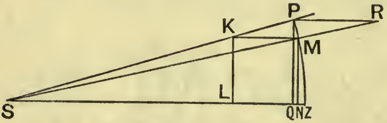

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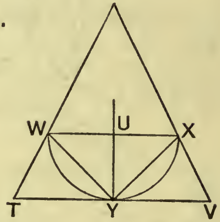

SIMILAR figures are equiangular and have their sides about their equal angles proportional. In the case of triangles, as we have already seen one fact is the consequence of the other, that is to say, if triangles are equiangular their sides about the equal angles are proportional and vice versâ. All regular figures of the same number of sides are similar. In a regular polygon all sides and angles are equal to one another. All circles are similar figures.

In other polygons the angles may be equal, but the sides are not therefore of necessity proportional.

To test whether two polygons are similar, we join all diagonals from one corresponding angular point in each.

If similar, they should now be divided into similar triangles corresponding each to each.

Let **ABCD**, **EFGH** be two unequal similar figures, so placed that corresponding sides are parallel, they are now said to be *similarly situated*, when we join corresponding points **A**, **E**;

B, F; D, H; C, G the straight lines obtained all meet at one point called a *centre of similitude.*

Now reverse the proposition, suppose the figure ABCD to be given and O the centre of similitude, suppose also that H is to be at a given distance from D, or on a given line, then join OA and produce it to meet HE, a parallel to DA, in E, this fixes the position of E. In the same way we can find all the angular points of EFGH. This method is used to inscribe in figures those similar to others, or to circumscribe figures about them.

It is not always necessary to draw the whole of both figures to determine sufficient points.

As an example suppose it is required to inscribe a square in sector PSZ to have two angular points on SZ, one on SP, and one on the arc PZ.

Draw PQ perpendicular to SZ, and PR parallel to it, and equal to PQ. Observe that PR and PQ are two sides of a square which can be completed by drawing a perpendicular from R to SZ produced, *and that this square has its sides parallel to the one required where* S *is a centre of similitude.* Join SR meeting the arc PZ in M, then M is one of the angular points required; draw MN perpendicular to SZ, and MK parallel to it; draw KL parallel to PQ or MN, then KMNL, is the square required. Since QPR is half a square, therefore LKM is half a square and the lines MN and LN parallel to KL and KM complete the square. The construction depends upon the fact that KL = KM because PQ = PR.

The centres of similitude of two circles are the points of intersection of their common external and internal tangents (figs. 65 and 66).

To inscribe semicircles in any figures, we find the point at which the semicircle is to touch, then draw a perpendicular there to the tangent, and bisect the right angles so obtained. For example to inscribe a semicircle in an isosceles triangle base TYU, bisect the base at Y, draw the perpendicular YU, *bisect the right angles* TYU *and* VYU, the bisectors meet the sides of the triangle in W and X, and WX is the diameter of the semicircle. It is sufficient to remember that the half right angle TYW determines the point W, then draw WX parallel to TU.

(81) Describe a triangle similar to a given one having its sides parallel to those of the given triangle, and an inscribed circle of given radius.

Let ABC be the given triangle. Find O the centre of the inscribed circle of ABC, then O is a "centre of similitude." With centre O and the given radius describe a circle. The tangents drawn to this circle parallel to AB, BC, CA form the required triangle DEF. The points of contact are found by drawing perpendiculars to the sides of ABC from O.

If with centre O we describe the inscribed circle of the triangle ABC, the triangles ABC and DEF with the circles form two completely similar figures with O as centre of similitude.

(82) Describe a triangle similar to a given one, having its sides parallel to those of the given triangle and a circumscribing circle of given radius.

Let ABC be the given triangle. Find O the centre of the circumscribing circle of ABC. Join OA, OB, OC, cutting the circle described with centre O and the given radius in D, E, F. DEF is the triangle required.

Compare this construction with 81, as before O the centre of similitude, but now the centre of circumscribing circle not of the inscribed circle.

(83) Describe a circle to touch two given straight lines, and pass through a given point.

Let OA, OB be the given straight lines, P the given point. Bisect the angle AOB by ODF, and take any point D on OF and describe with centre D a circle to touch OA and OB. Let E be one of the points at which OP meets this circle, we get a second solution by taking the other point at which OP meets it.

Draw PF parallel to ED to meet OF in F.

With centre F and distance FP describe the circle required.

O, the intersection of the common tangents to the two circles is a centre of similitude; ODE, OFP similar triangles.

(84) To inscribe a series of circles to touch two given straight lines, each circle touching the one before it in the series.

Let OA, OB be the two given straight lines, and suppose the first circle of the series described with given radius to touch OA and OB (71). Draw a tangent CD to the circle at the point where the bisector of the angle AOB meets it. Find the centre of the circle inscribed in the triangle OCD by bisecting the angle ODC, and describe the circle. Repeat the process as often as required.

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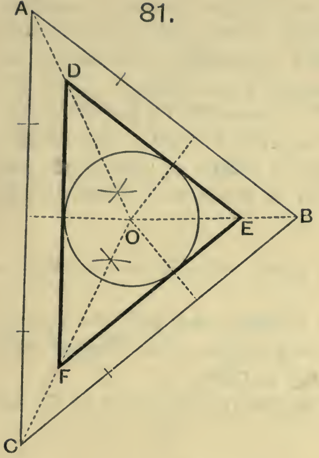

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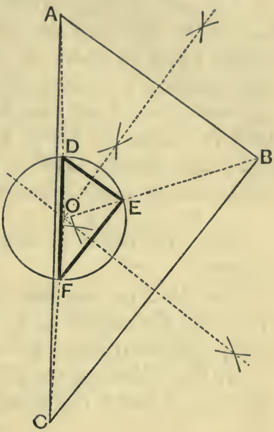

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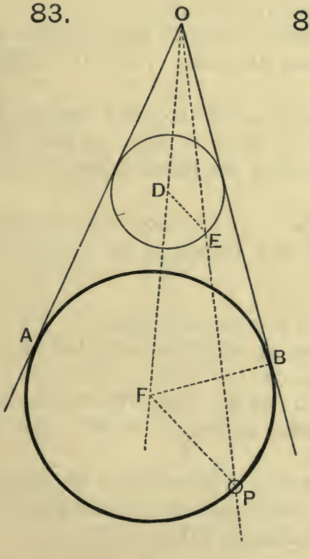

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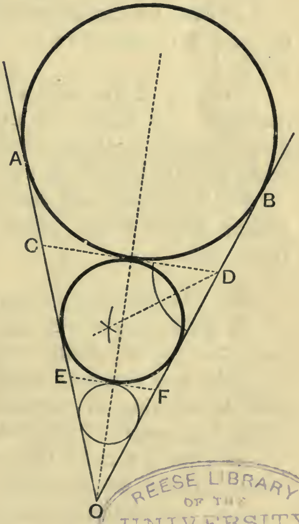

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(85) To describe in a given triangle one similar to another.

Let ABC be the given triangle. Draw LM (usually taken parallel to BC) and on LM make a triangle LNM having its angles equal to those of the second given triangle. Join AN and produce it to meet BC in F. Draw FD, FE parallel to NL and NM, then if we join DE it is parallel to LM. The triangle DFE is similar to LNM, A being a centre of similitude, and since the triangle LNM is similar to the second given triangle, DEF is the triangle required.

(86) To describe about a given triangle one similar to another.

Let ABC be the first given triangle, and DEF the second. Through the angles A, B, C draw parallels to the sides of DEF, this gives the triangle LMN similar to DEF.

If two sides are parallel in the two triangles such as EF and BC then B and N coincide.

(87) To inscribe a square in a given triangle.

Let ABC be the given triangle. Draw BK perpendicular to AC, and BD parallel to it. Make BD = BK. Join AD meeting BC in H.

H is one of the angular points of the required square, we complete the figure by drawing from H lines parallel and perpendicular to AC which determine the points G and E; then make EF = GH, and join FG.

A is the "centre of similitude." The method of finding this square EFGH is the same as that employed to find KLNM (page 46).

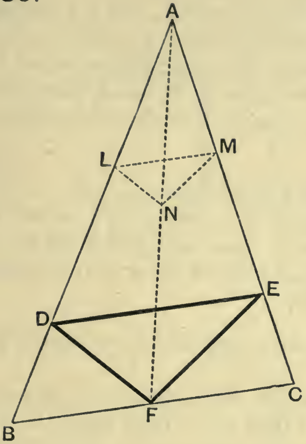
(88) To inscribe a square in any regular figure.

As an example take the regular pentagon described on the base CD (46) to inscribe a square in it.

Take two alternate angles B and E. Make the half square BEF by drawing EF perpendicular and equal to BE. Join AF using A as centre of similitude.

AF cuts a side of the pentagon in G which is an angular point of the square required. The square may be completed by drawing perpendiculars and parallels to CD as in (87).

85.



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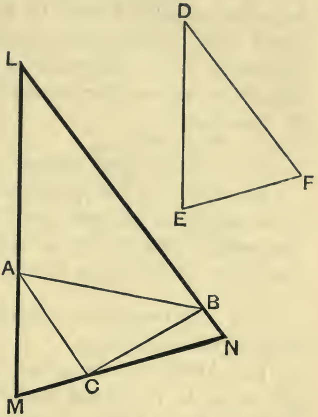

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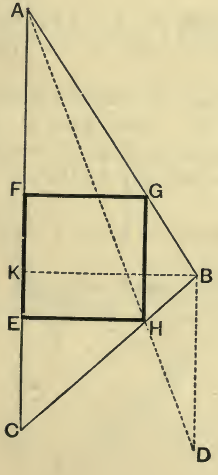

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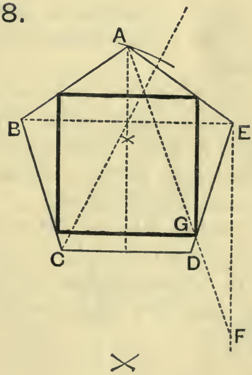

(89) To inscribe a square in a semicircle.

Let AB be the diameter of the given circle. Draw a straight line BD at right angles to AB, either above or below it, and equal to it. Take C the centre of the semicircle as centre of similitude. Let CD meet the semicircle in E, this is one of the angular points of the required square. Complete as in figures 87 and 88.

(90) To inscribe a square in a given sector.

Let CAB be the given sector. Join AB, and draw BD at right angles to AB and equal to it. Join CD meeting the arc of the section in E, this is an angular point of the required square. Complete as in figures 87 and 88.

(91) To inscribe an isosceles triangle in a given square, the base of which is given.

Let ABCD be the given square, AC a diagonal. Measure CH equal to the given base. Bisect CH at right angles by FG, then the figure FCGH is a square the diagonals of which bisect each other at right angles, so that FG = CH = the base required. Join AF, AG.

(92) To inscribe an isosceles triangle in a given square (or any other regular figure) when the vertical angle of the triangle is given.

Let AC be the diagonal of the square, (or the line bisecting any other regular figure drawn from A). Draw KAG at right angles to AC. With centre C and any convenient radius make a semicircle KLHG. Measure angles CAL and CAH each half the vertical angle given.

Produce AL and AH to meet the figure in E and F. AEF is the triangle required.

If the base angles of the isosceles triangle are given KAL and GAH are each made equal to one of the given base angles.

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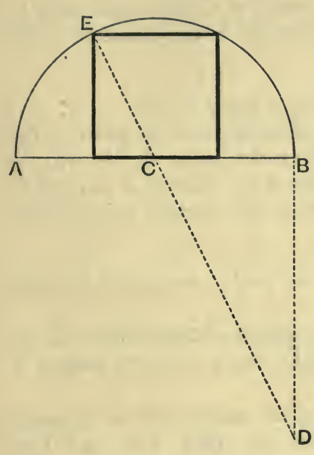

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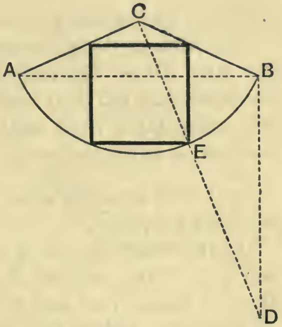

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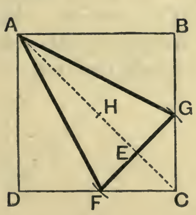

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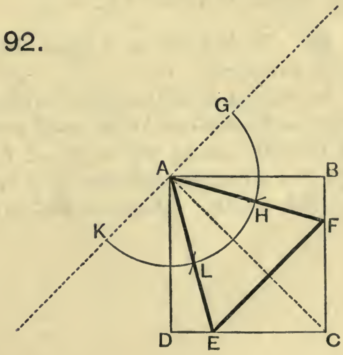

(93) To inscribe a semicircle in a given isosceles triangle.

This proposition has been already discussed on page 46. It will be remembered that we bisect the right angle ADB to find the point F, and FG parallel to BC is the diameter of the semicircle required.

(94) To inscribe a semicircle in a given sector.

Let ADE be the given sector. Find the middle point of the arc DE. Draw the tangent BC at the point C. It will be observed that BC is at once obtained by drawing a parallel to DE. Bisect the right angle ACB by the straight line CF, and proceed as in the last proposition.

(95) To describe about a given circle a triangle the angles of which are given.

In this case suppose two of the angles of the triangle are 60° and 45°. Take each of these angles from two right angles or 180°, we obtain 120° and 135°.

Draw the radius OD of the given circle and on opposite sides of it make the angles DOF and DOE, 120° and 135° respectively. Draw tangents at D, E, F forming the triangle ABC, this is the triangle required; for since DOF and DBF are together equal to 180°, and DOF = 120°, therefore DBF = 60°. In the same way we can shew that DCA = 45°.

The three angles of a triangle are together equal to two right angles therefore BAC = 75°.

(96) To inscribe in a given circle a triangle the angles of which are given.

At a point B of the given circle draw a tangent DBE (30). Make angles ABD and CBE equal to two of the given angles of the triangle cutting the circle at A and C. Join AC. Since angles in alternate segments of a circle are equal ABD = ACB and CBE = CAB. Therefore ABC is the triangle required.

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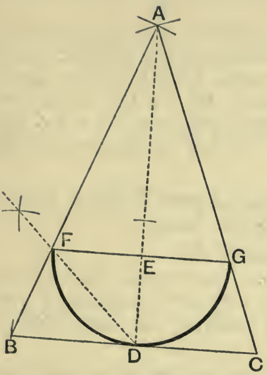

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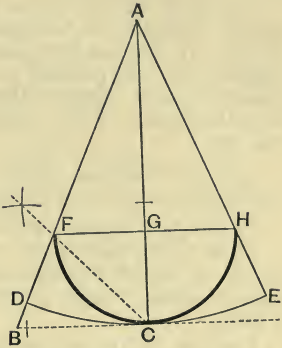

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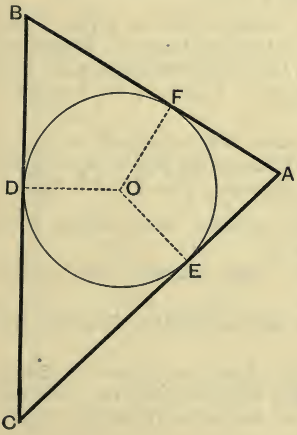

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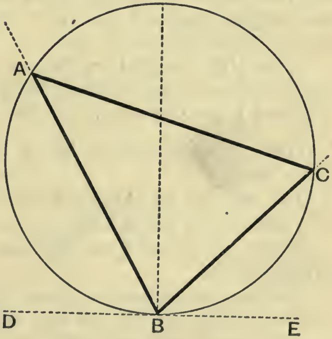


(97) To place a straight line of given length within a given angle with its ends of the lines bounding the angle, and parallel to a given straight line.

Let DAC be the given angle, and BC any straight line parallel to the given straight line, meeting AD and AC in D and C. In DC, or DC produced, mark off DE of the given length. Let EG parallel to AD meet AC in G. Draw FG parallel to DC. Since FGED is a parallelogram FG = DE, the length required. FG is parallel to DC and terminated by AD and AC.

(98) To describe within a regular figure another similar concentric one having its sides of given length.

In this case take a regular hexagon on AB.

Find O the centre of the figure. In regular figures the centre is found by bisecting two adjacent angles, or two adjacent sides at right angles, and producing the straight lines to meet.

Measure AC the side of the hexagon required along AB, and proceed as in the last proposition to find ED the base of the required hexagon. The inner hexagon is completed by joining O to the angular points of the given hexagon and describing a circle centre O and radius OE or OD to meet these lines.

(99) To describe about a regular figure another having a base of given length, and concentric with it.

In this case take an equilateral triangle ABC as given. The same method is used as in the last proposition, as before we find the centre of the figure but in this case produce CA to D of the length required, for the required figure is of necessity greater than the given one.

Draw DE parallel to OC to meet OA produced in E. A circle centre O, and radius OE will meet OB, and OC produced in G and F the remaining angles required. Join EF, FG, GE.

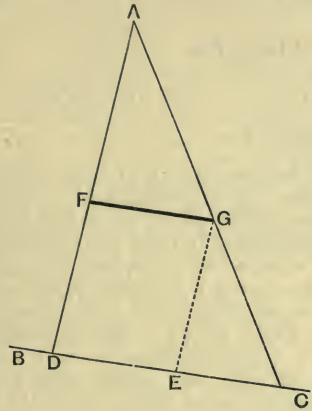
(100) To describe a figure on a given base similar to a given one.

Let ABCDE be the given figure. Join AD, AC, and measure AF equal to the given base.

Draw FG parallel to BC, then GH parallel to CD, next HK parallel to DE.

On the given base we can then describe a figure equal to AFGHK as in fig. 18, for the sides AF, FG, GH, HK, KA and the diagonals AG and AH can be measured from the figure AFGHK.

97.



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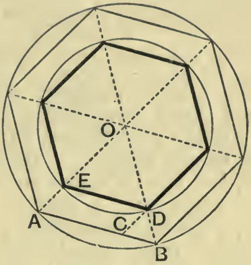

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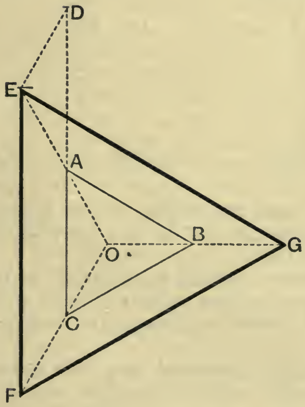

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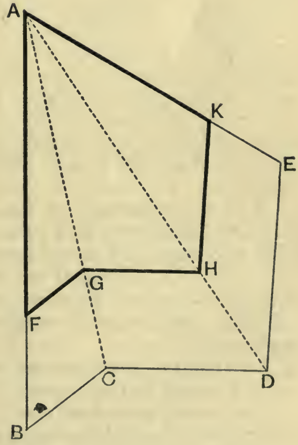
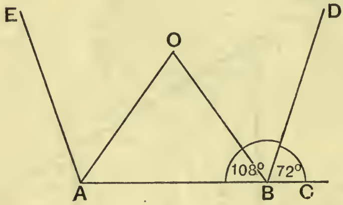


CHAPTER IX.

ON THE CONSTRUCTION OF REGULAR POLYGONS.

ALL regular polygons can be inscribed in a circle, and can have a circle inscribed in them. The definition of a regular polygon is "a rectilineal figure having all its sides and angles equal to one another." An equilateral triangle has three equal sides, and has its angles each equal to 60°; a square, four equal sides, angle 90°; a regular pentagon, five equal sides, angle 108°; a regular hexagon, six equal sides, angle 120°; a regular heptagon, seven equal sides, angle $128°\frac{4}{7}$; a regular octagon, eight equal sides, angle 135°; a regular nonagon, nine equal sides, angle 140°; a regular decagon, ten equal sides, angle 144°; a regular undecagon, eleven equal sides, angle $147°\frac{3}{11}$; a regular duodecagon, twelve equal sides, angle 150°.

1.



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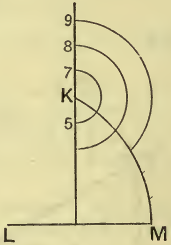

It is not necessary to remember the angles of the regular polygons, but learn this rule—Divide 360° by the number of sides and take the result from 180°.

This rule is evident if we observe that all the exterior angles of any polygon are together equal to four right angles or 360°, and that each exterior angle + the corresponding interior angle = 180°.

The central angle, or the angle subtended by a side at the centre is equal to the exterior angle.

The centre of the polygon is the same point as the centre of its circumscribing, or inscribed circle.

Therefore it is evident that if we know the centre of a polygon and its side in length and position we can find the figure, for we can describe the circumscribing circle at the distance of one of the given angular points, and step the side off round the circle.

As an illustration of the above rules take a regular pentagon (fig. 1), three of its sides EA, AB, BD are given in position and its centre O, which may be found by bisecting either the angles EAB, and ABD, or two of its sides at right angles.

Divide 360° by 5 we get 72° = the exterior angle CBD. The central angle AOB is also 72°. The angle ABD which is called the angle of the pentagon = 180° − 72° = 108.

There are several methods for finding regular polygons both in a given circle, and on a given base which are only approximate but are so near the truth that they are often used. As an illustration take figure 2.

On a straight line LM, find the vertex K, of an equilateral triangle, then we know that K is the centre of a regular hexagon on LM. With centre L and distance KM describe an arc, and by trial divide it into six equal parts. With centre K and at distances equal to the different points of division make arcs as shewn, numbering the points at which the arcs meet the perpendicular from K to LM as 5, 7, 8, 9......The points 5, K, 7, 8, 9......are the centres very approximately of regular polygons of 5, 6, 7, 8, 9...sides respectively.

As explained above we complete any one of the figures by the help of the circumscribing circle.

We have already had several special methods of describing regular polygons. An equilateral triangle on a base (5); a square (9); a square in a circle, observe that this is the same as (16), we draw two diameters at right angles, and join the points at which they meet the circle; a regular pentagon in a circle (45), and on a base (46); if we step off the radius of any circle round it the points so found are the angles of a regular hexagon inscribed in the circle (see page 10), and if we take alternate angles of the hexagon and join them we obtain an equilateral triangle in the circle; to describe a regular hexagon upon a given base, make an equilateral triangle upon it, with its vertex as centre describe the circumscribing circle of the hexagon, and step off the sides upon it.

If we bisect the arc subtended by the side of a square inscribed in a circle we find the side of a regular octagon inscribed in a circle by joining its ends. In the same way a regular decagon inscribed in a circle may be obtained from a pentagon, and a duodecagon from a hexagon.

To describe a regular polygon of any number of sides circumscribing a circle draw tangents at the angular points of a regular polygon of the same number of sides inscribed in the circle.

(101) To describe a regular heptagon in a circle by a special method.

Let BC bisect OA a radius of the given circle at right angles in D. BD is the length of the side of a regular heptagon inscribed in the circle.

This method is approximate but very nearly true.

(102) To describe a regular octagon in a circle.

Let B, A be the angular points of a square inscribed in the circle, centre O, found by drawing two diameters at right angles. Bisect the angle BOA, or which is the same thing the arc AB. Let the bisecting line meet the circle in C. Step off round the circle the sides of the octagon each equal to CA.

In the same way we find the side of a decagon when two angular points of a pentagon are given, or a duodecagon from a hexagon.

(103) To inscribe any regular polygon in a circle by a general method. In this case a regular pentagon.

Divide the diameter into as many equal parts as there are sides in the polygon.—OA into 5 equal parts.—Take B the *second* of these divisions, and join it to C the angular point of an equilateral triangle on OA.

Let CB meet the circle in L. OL is a side of the polygon.

This method is approximate.

(104) To inscribe any regular polygon in a circle by a general method.

Describe the given circle with centre O and radius OA.

Divide 360° by the number of sides, and thus obtain the number of degrees in the central angle.

At O make the angle AOC having the required number of degrees. Then AC is a side of the polygon.

The angle may be obtained by dividing the semicircle ACD, *by trial*, into the same number of equal parts that there are sides in the polygon, and take the *second* of these divisions from A. This will give the point C as before.

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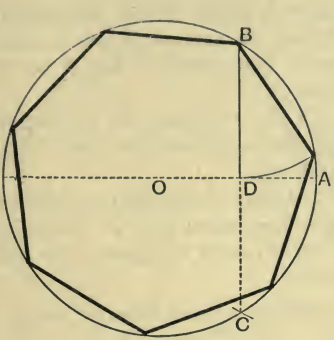

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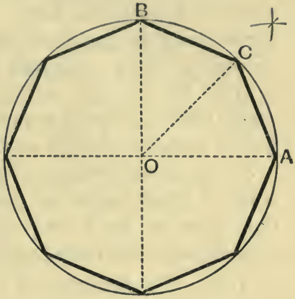

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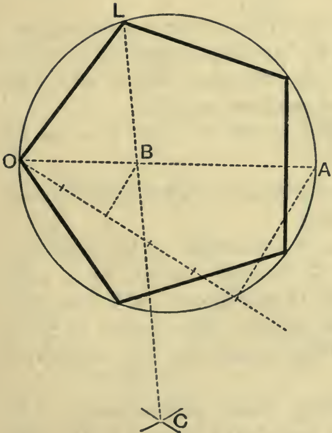

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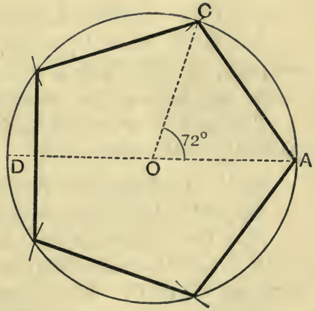

(105) To describe any given regular polygon on a given base.

Let AB be the given base, with centre B and distance BA describe a semicircle ADC. Divide the semicircle into as many equal parts as there are sides in the polygon. Let D be the second of these measured from C.

Join BD, this is a second side of the polygon, for the exterior angle DBC has been found by dividing twice 180°, that is 360° by the number of sides of the polygon.

To find the centre of the polygon bisect the two adjacent sides at right angles by straight lines meeting at O. Complete the figure by describing the circle centre O, distance OA, and step off the sides round the circle.

(106) To describe an octagon on a given base by a special method.

Let AB be the given base. Draw BE at right angles to AB, and bisect the exterior angle EBF by BC = AB.

Draw CD parallel to BE making CD = AB.

With centre D make an arc at distance AB to meet BE in E.

Describe the other half of the octagon in the same way commencing from A. If the drawing has been correct KE should be parallel to AB.

(107) To describe a regular polygon about a given circle.

Let G, C, A, B, H, K be the angular points of a regular polygon, in this case a hexagon, described within the given circle and having the required number of sides.

Through A draw DE parallel to BC, through B draw EM parallel to AH, and so on.

(108) To describe a regular octagon within a given square.

Let AEHK be the given square.

Join the diagonals AH and KE meeting at the centre O. With centre E and distance O describe an arc of a circle meeting AE in D. With centre O and distance OD describe a circle cutting the sides of the square. The points at which this circle meets the sides of the square are the angles of a regular octagon.

It will be found that if OB be drawn perpendicular to AE then OD bisects the half right angle AOB, so that COD = DOF = double DOB = 45° = the central angle of a regular octagon.

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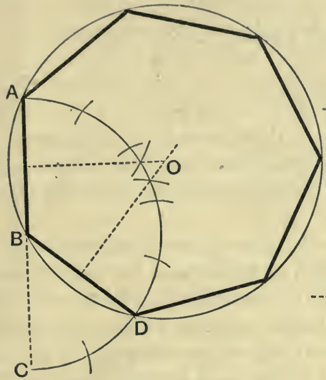

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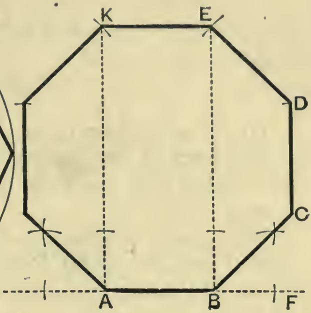

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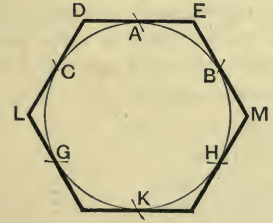

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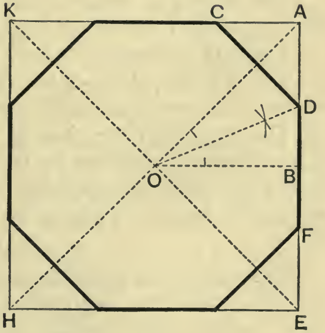


CHAPTER X.

SCALES.

When any plane figure is represented by a drawing, all straight lines in the figure should bear the same proportion to one another that the lines corresponding to them do in the drawing.

Therefore if we know the length of one straight line in the drawing and the actual length it represents we can by comparison find the actual length represented by any other straight line in the drawing. For example suppose we have the plan of the floor of a room before us and we measure its length and find it 4 inches, its breadth 3 inches, and its diagonal 5 inches, and if we were told that the actual length of the room is 20 feet, then the breadth is 15 feet and its diagonal 25 feet. We arrive at the result as follows, we know that 4 inches represents 20 feet, that is 240 inches, therefore one inch on the plan represents 60 inches actual distance.

We infer that every other straight line measured on the plan represents an actual distance of 60 times as much.

The plan is drawn to a scale $\frac{1}{60}$ of the original.

This fraction, $\frac{1}{60}$, is called the REPRESENTATIVE FRACTION of the scale sometimes represented by the letters R. F.

The rule then for finding the representative fraction is this—Find any distance in inches on the plan, or map, and divide by the corresponding distance actually shewn also *reduced to inches*. Thus in the example above

$$\text{R. F.} = \frac{4}{240} = \frac{1}{60}.$$

The rule is best remembered in the form

$$\text{R. F.} = \frac{\text{Map distance in inches}}{\text{Real distance in inches}}.$$

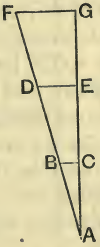
Hence we draw the following inferences.

When we know the R. F. and the real distance we find the corresponding map distance by reducing the real distance to inches and multiplying by the R.F., or which is the same thing if the numerator of the R.F. is 1 divide the real distance, when reduced to inches, by the denominator of the R.F.

Suppose a straight line of 3 inches represents 20 feet and a scale is required to measure 50 feet: we take the representative fraction $=\frac{3}{240}=\frac{1}{80}$. Divide 50 feet by 80 we obtain $7\frac{1}{2}$ inches, the length of the scale.

We could also have obtained the same result by remembering that 20 feet has to 50 feet the proportion of 2 to 5 and have found a fourth proportional (39) to the numbers 2, 5, 3.

If the map distance and R.F. are given to find the real distance divide the map distance by the R.F. As an example suppose the R.F. is $\frac{1}{60}$ and the map distance 5 inches. Divide 5 inches by $\frac{1}{60}$ or which is the same thing multiply by 60 we obtain 300 inches $=$ 25 feet.



The proposition upon which the diagonal scale depends is that if we divide two sides of a triangle into any number of equal parts by straight lines parallel to the base each of these parallels is greater in length than the one before it by the same amount, which is the length of the least parallel.

For example suppose the sides of the triangle AFG are divided into three equal parts by DE and BC parallel to the base FG, then BC $=\frac{1}{3}$ of FG, DE $=\frac{2}{3}$ of FG. Similarly if AF and AG were divided into ten equal parts by straight lines parallel to the base FG these lines would be respectively $\frac{1}{10}$, $\frac{2}{10}$, $\frac{3}{10}$, $\frac{4}{10}$, $\frac{5}{10}$, $\frac{6}{10}$, $\frac{7}{10}$, $\frac{8}{10}$, $\frac{9}{10}$ of FG.

When two different scales are used for the same map, for instance an English scale of inches and a French scale of centimetres, they are said to be comparative. All we need remember is that comparative scales have the same representative fraction.

(109) To draw a plain scale.

In a certain map 2 miles are represented by $2\frac{1}{5}$ inches, draw a plain scale to measure up to 5 miles, shewing furlongs.

We can find the distance that represents 5 miles by the arithmetical rules just given or by finding a fourth proportional to 2, $2\frac{1}{5}$, 5 (39).

By arithmetic the working is as follows: the R.F. is $2\frac{1}{5}$ inches divided by 2 miles reduced to inches

$$=\frac{11}{5 \times 2 \times 1760 \times 36}=\frac{1}{57600}.$$

The number of inches in 5 miles $= 5 \times 1760 \times 36$. Divide this by 57600 we get $5\frac{1}{2}$ inches.

Divide this straight line into 5 equal parts, each of these parts represents one mile. Take the last division on the left, and on the right of it put THE ZERO POINT. Divide this last division into eight equal parts to shew furlongs.

In drawing the scale first make the lower line moderately thick, and of the length required, then make two perpendiculars at the ends about the same length as shewn in the example. Next draw a thin line parallel to the first and terminated by the perpendiculars, and bisecting them, or nearly so.

The perpendiculars drawn at the points of division marking miles should be drawn equal to those at the ends of the line, but those marking furlongs only to meet the thin line.

In figuring the scale observe that *the miles* read *from the zero point to the right*, and the *furlongs*, *from the zero point to the left.*

To measure a distance from the scale, for example 3 miles 5 furlongs, we measure from the 3 of the miles to the 5 of the furlongs, as marked by asterisks.

(110) To draw a diagonal scale.

In a certain map 2 miles are represented by $2\frac{1}{5}$ miles, draw a diagonal scale to measure miles, furlongs, and chains, any distance up to 5 miles.

As in the last example we find the length required to represent 5 miles is $5\frac{1}{2}$ inches. This distance is, as before, divided into 5 equal parts, and the last division on the left-hand side into 8 equal parts to shew furlongs.

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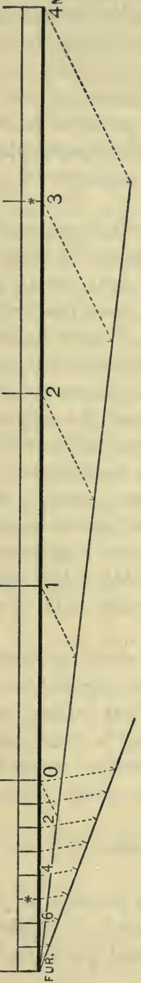

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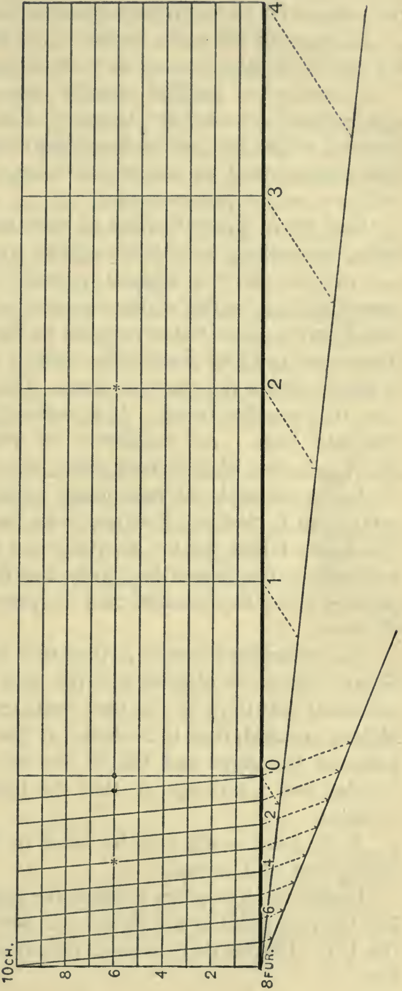


110.

The zero point occupies the same position and the figuring of the scale as far as regards the miles and furlongs is also the same.

In drawing the scale, however, the perpendiculars drawn at the end of the first line are now an inch or more in length.

A number of parallel straight lines are now drawn to the first line, in this case ten, terminated by the perpendiculars at the end of the line, all at the same distance from one another. This can be done by marquoise scales, or by marking off equal distances on the perpendiculars.

Now draw perpendiculars at each of the divisions marking miles, by making parallels to those at the ends. Divide the last division on the highest parallel into eight equal parts corresponding to the eight divisions on the base line (or first line drawn). Join the zero point to the first of these divisions, the first on the base line to the second on the highest line, and so on, to obtain the slanting lines. The parallels are numbered 1 to 10 from the lowest. It is sufficient to put the figures on alternate lines. As explained on page 65 we thus obtain $\frac{1}{10}$, $\frac{2}{10}$, $\frac{3}{10}$, &c. of 1 furlong, that is 1, 2, 3, 4...chains.

As an example of measuring any distance, take 2 miles 3 furlongs 6 chains. Starting from the base line division for 3 furlongs follow up the slanting line to where it crosses the parallel for 6 chains, this gives the first point. The second point is where this parallel cuts the perpendicular at the 2 miles division.

To understand how it is that this length does measure the distance given we observe that the part on the parallel between the small dots is $\frac{6}{10}$ of the part representing one furlong on the highest parallel, that is 6 chains, it can easily be seen that the parts on the right and left of the dots represent respectively 2 miles and 3 furlongs, so that the total is 2 miles, 3 furlongs, 6 chains.

A diagonal scale can be used in the same way to shew yards, feet and inches.

Divide the line given to shew the required number of yards. Put the zero point at the right-hand side of the last division on the left. Divide this division into three equal parts to shew feet.

Let the figuring always run to the right and left of the zero point, always reading from the zero point.

Complete the drawing as before, but in this case draw 12 parallels to the base (not 10) to shew inches.

To measure then 3 yards, 2 feet, 7 inches.

Find the point indicating 2 feet, follow the slanting line from it to the seventh parallel, (for the seven inches,) and follow along this parallel to where it crosses the perpendicular for 3 yards, this gives the second point.

The diagonal scale on the protractor giving inches, or half inches to hundredth parts is that most used for drawing where inches and decimals of inches are required.

Take the diagonal scale representing inches and hundredths of inches. The base line is divided into a certain number of inches, the last division on the left into ten equal parts shewing tenths of inches, the zero point is on the right of this division, and there are ten parallels dividing each tenth into ten parts, that is into hundredths.

To measure 3·42 inches. Find the point indicating ·4 or $\frac{4}{10}$ of an inch, follow along the slanting line drawn from this point to where it meets the second parallel to shew $\frac{2}{100}$, the second point on which to place the compass is where this second parallel meets the perpendicular through 3 inches.

(111) On the use of the scale of chords.

On the protractor or ruler will be found a scale marked C.H.O. figured with unequal divisions from 0 to 90. This scale indicates the lengths of the chords subtending the number of degrees marked, at the centre of a certain circle.

To find this circle we remember that the radius subtends an angle of 60° at the centre, therefore the chord of 60° is equal to the radius.

To make a right angle at the point O in the straight line OA, describe a circle with centre O and radius equal to the chord of 60° from the scale. Place in this circle the chord of the scale marked 90, that is the whole scale, represented by AB, then $BOA = 90°$. Similarly if we place in the circle chords $AC =$ chord 60°, or $AD =$ chord 40°, we obtain angles of 60° and 40° at the centre O.

(112) To construct a scale of chords.

We may either be asked to construct the scale on a given straight line as AB, or of a given radius as OA.

If AB be given, bisect it at right angles and draw a perpendicular equal to half of AB, this gives the point O. With centre O describe the arc AB.

If OA be given describe a circle with any centre O and radius OA and draw two radii OA, OB at right angles, and join AB.

With centres A and B and distances AO and BO make arcs to cut the arc AB in the points marked 30° and 60°.

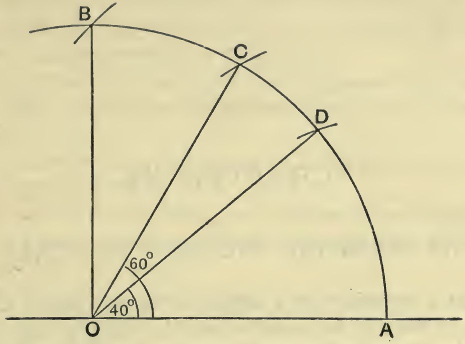
Divide each of these arcs into three equal parts by trial.

With centre A and distances respectively equal to the points of division mark off the chords of the different angles on the straight line AB.

If it be required to measure angles up to 5° we must divide the whole arc AB into 18 instead of 9 equal divisions. This is done by dividing the arc AB into 9 equal divisions, bisecting one of them and marking off the middle points of each of the other arcs by the distance thus obtained.

The scale is completed as far as regards drawing in the same way as the plain scale: in figuring however the zero point is at the end of the scale.

111.



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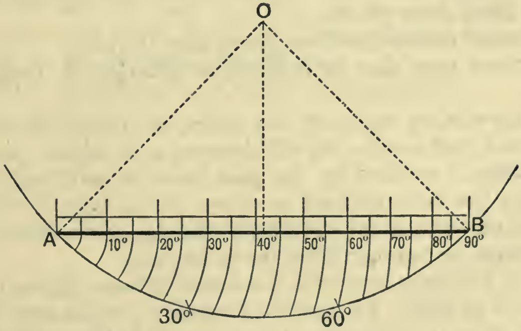


CHAPTER XI.

ON DRAWING FIGURES TO SCALE.

When a drawing of a figure is to be copied it may be required to draw it accurately from the copy, or else it may be roughly drawn and numbers written upon the sides of the figure shewing the lengths of the lines.

Figures may be copied or reduced by the method of squares.

A piece of glass with squares of a certain size ruled upon it is laid over the figure. Another set of squares is ruled upon the paper upon which the copy is to be made. By observing the points at which the sides of the squares on the glass cut the different lines of the figure on the plan, we can mark corresponding points on the squares on the copy and fill in the figure from these points.

Another method has been given, page 56 (100).

Figures may also be reduced, or enlarged by proportion (113).

Otherwise we may draw two scales, one to suit the plan to be copied, and another for the drawing to be made. Measure any distance required on the plan from its own scale, then measure the same distance as shewn on the second scale.

By constructing any convenient triangles on the drawing, by this means we can then fill in the figure (114).

It is necessary to draw a distinction between figures copied to size or to scale. For example suppose a triangle were drawn to half the scale of another, it would not be half the area of the first but a quarter of it. Draw an equilateral triangle on a base of two inches, one on a base of one inch is quarter of it in area, but one drawn on 1·41 inches is about half of it in area. A rule will be given in the next chapter for reducing or enlarging figures in a given proportion of area.

When a figure is to be copied, that is symmetrical in form, it is quite sufficient to give the measurement of very few lines in order to describe it.

Many constructions depend upon circular arcs, the centres of which must be found.

If two circular arcs form one continuous curve the centres of both circles are in a straight line passing through the junction of the two curves.

When the point of junction is between the centres the curve either takes the form of a cusp, or an S-shaped curve called an ogee (arc FCE, 155).

When centres are on the same side of the point of junction the curve continues in the same direction as shewn in spirals composed of circular arcs (136) and ovals (141).

The rule then is in both cases, when the centres are given join them and find the point of junction on this line: but when the point of junction and one centre is given join them and find the second centre on this line.

The cusp above mentioned is one formed by tangential arcs, as in the trefoil of tangential arcs in (139). Arcs are not always tangential (see 133 and 134) but the test for their being tangential is as stated above that the centres and point of junction are in the same straight line.

Definitions of foils, &c. are given in Chapter XV.

(113) ABCED is a given figure, it is required to construct one similar to it, the line *ac* corresponding to AC being given in length and position. A and B are taken as centres for the arcs CE and DE.

Draw any two straight lines OL and O*l* at any convenient angle.

Measure OL = AC, ON = BC, OM = AB, observing that BD = AC, and AD = BC.

Measure O*l* = *ac*.

Join L*l* and draw M*m*, N*n* parallel to L*l*. Then O*l* is divided similarly to OL.

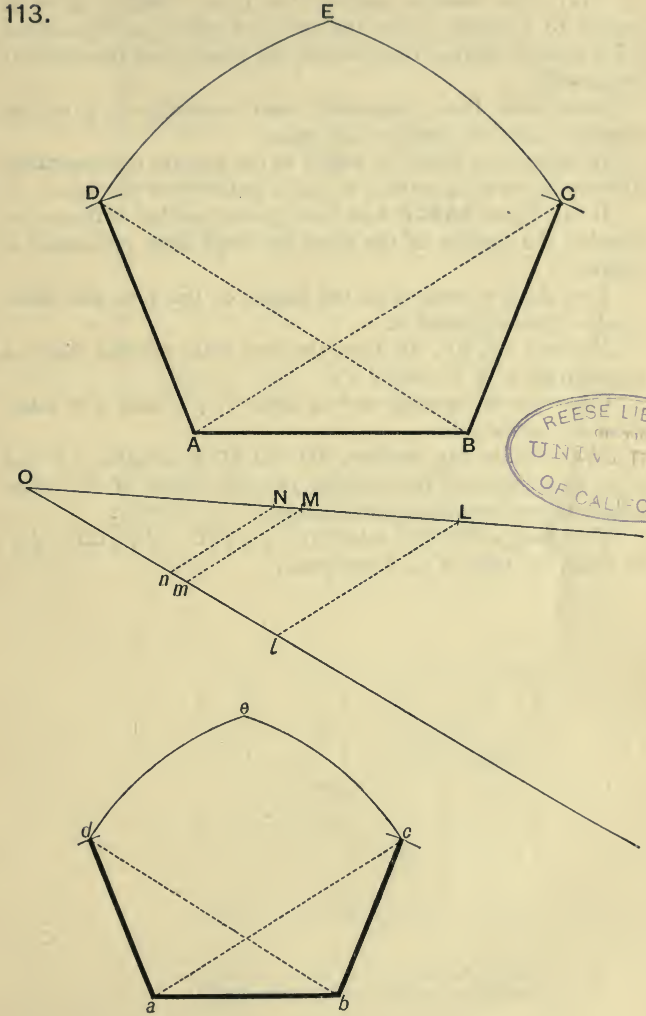
With centre *a* and distance O*m* = *ab*, and with centre *c* and distance O*n* = *bc* make arcs to meet at *b*. Join *ab* and *bc*. Next on the base *ab* construct the triangle *dab* where *ad* = *bc* and *bd* = *ac*. With centre *a* and distance *ac* make the arc *ce* and with centre *b* and distance *bd* make the arc *de*.

abced is the figure required.

When it is required to reduce a figure by the method of squares, we take a piece of glass on which squares are ruled, and place it on the plan in such a way as to get as many angles and lines on chief points of the figure.

Then rule on the paper lines at such a distance apart as to make squares of the required size.

113.



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(114) For example suppose the figure EABCD is to be copied to $\frac{2}{3}$ scale. Place the piece of glass having squares ruled upon it so that the lines on the glass cross the figure as indicated.

Next draw lines horizontally and vertically at $\frac{2}{3}$ of the distance apart of those on the glass.

In the second figure at points in the squares corresponding to those in the first, mark *a*, *b*, *c*, *d*, *e* and draw in the figure.

If the figure EABCD had been given together with a scale shewing the lengths of the sides we could have proceeded as follows.

First draw a scale $\frac{2}{3}$ of the length of the first and make similar divisions upon it.

Measure EA, EB, AB from the first scale we find they are respectively 2′, 4′ 3″, and 2′ 7″.

Construct the triangle *eab* of sides 2′, 4′ 3″ and 2′ 7″ taken *from the second scale.*

In the same way measure BC and EC of lengths 1′ 3″ and 5′ 4″ and construct the triangle *ebc* with sides of the same length taken from the second scale.

The triangle ECD has sides EC = 5′ 4″, CD = 2′ 7″, ED = 4′ 2″ to which the sides of *ecd* correspond.

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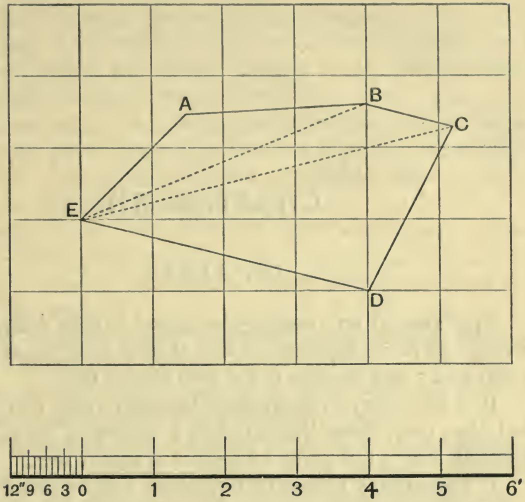

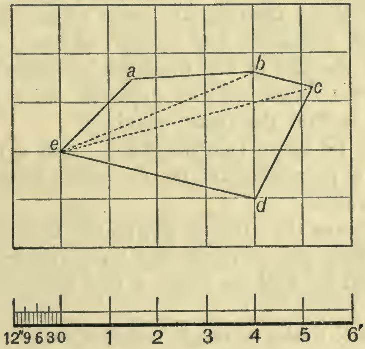

CHAPTER XII.

ON AREAS.

THE area of any rectangular figure can be found by multiplying two of its adjacent edges : thus if a rectangle have sides 2 feet and 3 feet its area is $2 \times 3 = 6$ square feet.

It is necessary to distinguish between linear feet and square feet, thus twice three linear feet is a length of 6 feet, but 2 feet multiplied by 3 feet gives an area of 6 square feet.

If we multiply square feet by linear feet we obtain cubic feet so that 6 square feet multiplied by 2 linear feet gives the volume or space occupied by a block the area of the base of which is 6 square feet and of 2 feet thickness.

In the same way inches multiplied by inches give square inches, and square inches by linear give cubic inches.

In thinking then of the areas of any figures we compare them with rectangles and squares of equal area, because when we know the size of the rectangle, or square of equal area we only have to measure two adjacent sides and multiply them together to find the result required.

It will be found (Euclid, Bk. I.) that a triangle is half the area of a parallelogram on the same base and between the same parallels.

To obtain the area of a triangle then we draw a perpendicular from the vertex to the base and multiply the altitude thus found by *half* the base, or else the *whole* base by *half* the altitude.

Again to find a rectangle equal in area to a triangle we make its sides equal to the base and half the altitude of the triangle, or else to the altitude and half the base.

To make a triangle equal to a parallelogram we draw it on the same base but double its altitude, thus if ABCD be a parallelogram and if we produce BA to E making AE = AB then the triangle EBC is equal in area to ABCD.

It is proved in (Euclid, Bk. I.) that triangles upon the same base and between the same parallels are equal to one another, by means of this proposition any rectilineal figure can be reduced to a triangle of equal area.

A triangle can be reduced to an isosceles triangle, or one having any given angle.

Take any triangle ABC and through its vertex draw AD parallel to the base BC.

Bisect BC at right angles by a straight line meeting AD in E, then EBC is an isosceles triangle equal to ABC.

If we draw BM making any given angle with BC meeting AD in M, the triangle MBC is also equal to ABC.

If on BC we make the segment of a circle to contain any given angle (32) and this segment meets AD in P or Q, the triangles PBC and QBC will each have the given vertical angle, and be equal in area to ABC.

When therefore the area and base of any triangle are given to construct it, make a rectangle on the given base of the given area; make a triangle of the same area by doubling the height of the rectangle; through the vertex of this triangle draw a parallel to the base. The vertex of the triangle required is somewhere on this parallel. For example suppose we want to describe a triangle on a base of 2 inches, the area of which is 4 square inches, and one side 5 inches long.

Draw the base of 2 inches, and a parallel to it at a distance of 4 inches, from one of the ends of the base as centre and at a distance of 5 inches make an arc to cut the parallel to the base, this point is the vertex of the triangle required.

When four straight lines are in proportion (Chap. III.) the product of the extremes is equal to the product of the means, thus 2 : 3 : : 4 : 6, then 2×6 the product of the means $= 12$, and 4×3 the product of the means is also 12.

This property is very useful in propositions relating to areas for it follows that when four straight lines are proportionals the area of the rectangle contained by the extremes is equal to that contained by the means. For instance suppose we had a rectangle the sides of which were 3″ and 4″ but wanted to make one equal in area to it on a base of 5″. Find a fourth proportional to 5″, 3″, and 4″, namely $2\frac{2}{5}$″, then the rectangle required has sides 5″ and $2\frac{2}{5}$″ for on multiplying these together we get 12 square inches $= 4'' \times 3''$.

A special case of this proposition is that of the third proportional. When three straight lines are proportionals, the product of the extremes is equal to the square on the mean. Thus $3 : 6 :: 6 : 12$. Then $3 \times 12 = 6 \times 6$, or the product of 3 and 12 = the square on 6. Hence it follows that to find a square of given area we make any rectangle of that area, and find a mean proportional between its sides.

If the square be given to find a rectangle of the same area on a given base, find a third proportional to the given base and the side of the square, which is equal to the remaining side of the rectangle.

Thus to find on a base 12 a rectangle equal to a square on 6, we find the third proportional to 12 and 6, $12 : 6 :: 6 : 3$ then 3 is the other side of the rectangle.

To make a figure equal in area to the sum of two similar figures on bases AB and CD, place lengths equal to AB and CD at right angles to form the sides of a right-angled triangle.

The hypotenuse is the base of a similar figure of area equal to the sum of the other two.

To make a figure equal in area to the difference of two similar figures on bases AB and CD, AB being the greater construct a right-angled triangle hypotenuse = AB and one side = CD, the other side is the base of the figure required (27).

As circles are similar figures the same rules apply if we use radii instead of bases.

To find approximately a square equal in area to a given circle, divide its diameter into nine equal parts and describe a square on a side equal to eight of them.

The side of the square thus found is too large by about $\frac{1}{30}$ part of one of the divisions.

[To find more exactly the error multiply 81 by 3·141592 and extract the square root, the result is 7·9......, shewing that by taking 8 of 9 parts we take too much.]

When four straight lines are proportionals and similar figures are described upon the first and second, and squares upon the third and fourth. Then if the similar figure upon the first is equal to the square upon the third, the figure upon the second is equal to the square upon the fourth. Thus to make a regular pentagon the area of which is 10 square inches. Make any isosceles triangle vertical angle 72° on a base AB. Make a square equal in area to it by finding the mean proportional between the altitude and half the base, let the side of this square be CD; and let EF be the side of a square of area 2 square inches, that is a mean proportional between 1″ and 2″.

Then a fourth proportional to CD, EF, AB is the base of the pentagon required, for it is the base of an isosceles triangle of vertical angle 72° of area 2 square inches, and five of these make up the pentagon which will be of 10 square inches in area.

(115) To reduce the figure ABCDEF to a triangle of equal area having its base in AF and its vertex at D.

Produce AF both ways.

Count the points first, second, third, namely A, B, C. Join the first and third, A and C ; and draw a parallel through B the second point to meet AF in G. Join CG, then the figure GCDEF is equal to ABCDEF. The one side CG has replaced AB and BC, because the triangles ABC, GAC are equal, being between the same parallels. Now repeat the process, count G, C, D, as first, second, third point; draw CH parallel to DG, then DH replaces GC and CD. Draw EK parallel to DF, then DK replaces FE and ED. We now get finally the triangle DKH = the figure ABCDEF.

Any triangle having its base in the line FH and equal to KH and its vertex in a parallel through D will be of the required area. For example, draw KX at right angles to HK meeting the parallel through D in X, then XKH is a right-angled triangle equal in area to ABCDEF. Therefore the area ABCDEF $=\frac{1}{2}$KX × KH.

(116) To make one figure similar to another and bearing a given proportion to it in area.

Let ABCDE be a given figure it is required to make one similar to it and $\frac{2}{3}$ of it in area. Make AF $=\frac{2}{3}$ of AB by dividing AB into three equal parts and making AF equal to two of them.

Find AX a mean proportional between AF and AB (42). Then A*b* made equal to AX is the base required. Complete the figure as in (100). To enlarge the figure say in proportion of 4 to 3. Make AF $=\frac{4}{3}$ of AB by dividing AB into three equal parts and taking 4 of them. Find the mean proportional as before. A*b* will now be greater than AB, and AC, AD, AE must be produced before drawing parallels.

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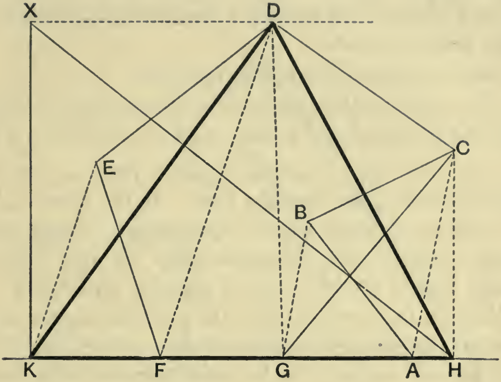

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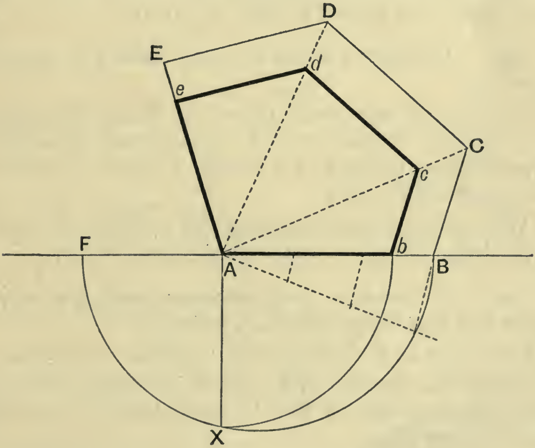

(117) To find the side of a square equal in area to a triangle ABC.

Draw AD perpendicular to BC, and produce it to a length $=\frac{1}{2}$BC. Find the mean proportional between the altitude and half the base by making a semicircle on this line to cut BC or BC produced in X.

DX = the side of the square required.

The square can be described by measuring off a distance = DX along AD produced and making arcs to intersect in F (9).

(118) To make a square equal in area to the sum of the squares on any given straight lines. In this case 1″, $\frac{1}{2}$″, $1\frac{1}{2}$″.

Draw two straight lines at right angles. Along one of them measure OB = 1″ and along the other OA = $\frac{1}{2}$″, then (Euclid I. 47) the square on AB = sum of squares on OA, OB.

Measure OC = AB and OD = $1\frac{1}{2}$″ then the square on DC = sum of squares on OC, OD = sum of squares on OA, OB, OD.

If another square is to be added measure off a line equal to DC along OC, and the other given line along OD, and join their ends; and so on for any number of squares.

(119) To divide a circle into any number of annuli of the same area.

In this case take three annuli, OA being the radius of the given circle. Divide OA into three equal parts and draw perpendiculars to OA at the points of section to meet a semicircle on OA at D and E.

With centre O and distances OD and OE describe circles. These divide the given circle into the annuli required.

(120) To divide a circle into any number of equal parts which are also all of the same perimeter.

[Perimeter or periphery means "measure all round."]

Divide the diameter of the circle into the number of equal parts required, and on the divisions describe semicircles as shewn in the figure.

In this case three parts were taken, and semicircles described on AC, CB and on AD, DB.

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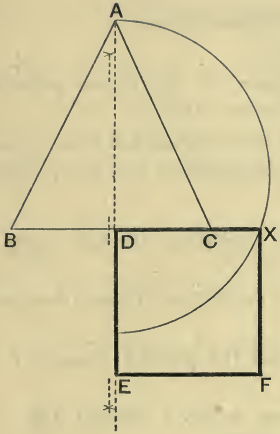

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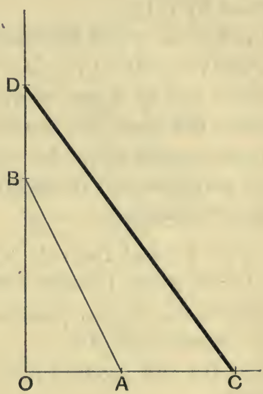

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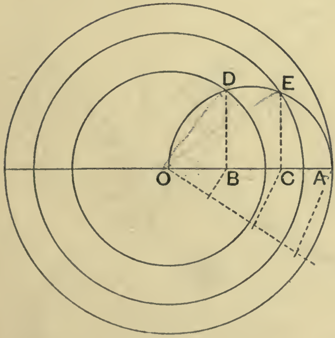

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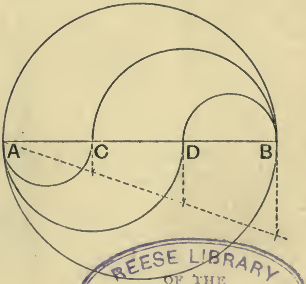

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(121) To make a figure similar to LNMS and equal in area to ABCD.

Reduce ABCD and LNMS to squares of equal area (115) followed by (117).

Let EX and PY be the sides of these squares.

Take YP : NM : : Y*p* : *nm* (39).

Then *nm* is equal to the base of the figure required. Measure MR=*mn* and proceed as in (100).

In the second figure the mean proportional has been worked on the perpendicular through the bisection of the base, instead of on the altitude.

(122) To find the side of an equilateral triangle equal in area to the given triangle ABC.

Find D the angular point of an equilateral triangle described on the shortest side AB.

Join DA and produce it to meet the parallel through C to AB, in E.

Find AX the mean proportional between AE and AD. On AX describe the equilateral triangle AXY.

The triangle AXY is equal in area to ABC.

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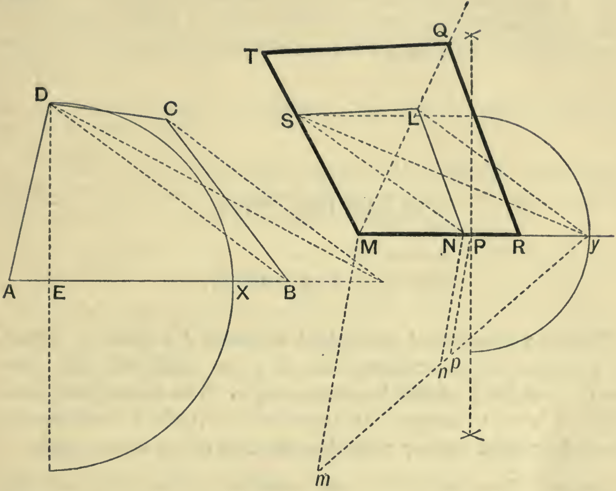

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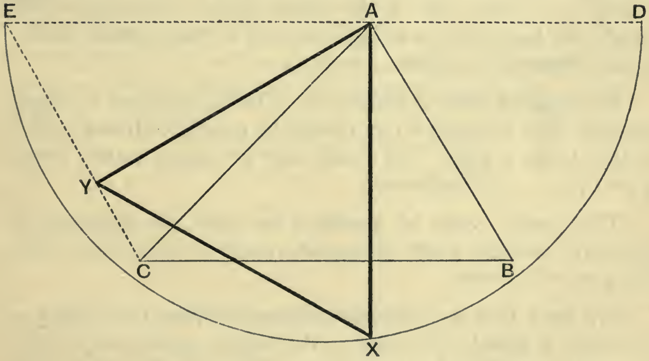

CHAPTER XIII.

ON SQUARE ROOT.

WHEN a number is multiplied by itself the result is called the square of that number, thus if 7 be multiplied by 7 we obtain 49 which is called the square of 7. The reason the word square is used to denote this process is obvious, it is because a square described upon 7 units has an area of 49 square units.

Suppose however the square number to be given and the number required of which it is the square. This number is called the square root of the given square. Sometimes the result can be exactly obtained, thus if 81 were given as the square number its square root is 9.

But suppose such a number as 3 to be given as a square number, then its square root cannot be exactly obtained, and it is said to be a surd. The result may be approximately found $\sqrt{3}=1{\cdot}732\ldots\ldots$ by arithmetic.

The square roots of numbers can also be obtained by geometry and the result measured exactly to within less than $\frac{1}{100}$ part of an inch.

We know that the mean proportional between the lengths of two lines is equal to the side of the square equal in area to the rectangle contained by these lines. Therefore the side of the square can be measured and is the square root of the product of the numbers representing the two given lines.

$\sqrt{3}$ is the mean proportional between its factors, namely 3 and 1; $\sqrt{6}$ is the mean proportional between 2 and 3; $\sqrt{12}$, between 3 and 4; $\sqrt{\frac{3}{2}}$ is the mean proportional between $\frac{3}{2}$ and 1, or between 3 and $\frac{1}{2}$. *The square root of a number is the mean proportional between two factors, which multiplied together give the number.*

The square roots of numbers can also be obtained by the use of Euclid I. 47.

If ABC be a right-angled triangle, A being the right angle, the square on BC = the sum of the squares on BA, CA; and the square on BA = the square on BC − the square on CA.

We can then by use of this proposition find the square root of any number which is the sum or difference of two squares.

For example to find the square root of 45; 45 is the sum of 36 and 9, if then we make a right-angled triangle the sides containing the right angle measuring 6″ and 3″, the square on the hypotenuse $= 6^2 + 3^2 = 45$ square inches, therefore the hypotenuse $= \sqrt{45}''$.

To find $\sqrt{27}$ we observe that $27 = 36 - 9$. On hypotenuse 6″ make a right-angled triangle, one of the other sides being 3″, then the square on the remaining side $= 36 - 9 = 27$ square inches. The length then of this third side $= \sqrt{27}''$.

In finding square root it is usual to take inches as the unit to work by, but when the numbers are large half inches are taken, for on most protractors hundredth parts of half inches are given.

[*The drawings on page* 91 *are to half scale.*]

(123) To find $\sqrt{6}$.

Find the mean proportional between 3″ and 2″. Measure the length of this line we find it about 2·45″.

Therefore we take $\sqrt{6}=2{\cdot}45$.

(124) To find $\sqrt{5}$.

We can take $\sqrt{5}$ as the mean proportional between 5 and 1, or else as in the figure make a right-angled triangle of sides 2″ and 1″, then the area of the square on the hypotenuse $=2^2+1^2$ in square inches $=5$ square inches. Therefore the length of the hypotenuse $=\sqrt{5}''=2{\cdot}23''$.

(125) To find $\sqrt{8}$.

$\sqrt{8}=$ the mean proportional between 4 and 2, otherwise 8 is the difference between 9 and 1. Make a right-angled triangle hypotenuse 3″ and one side 1″, then the remaining side measures $\sqrt{8}''=2{\cdot}83''$.

(126) Another method of finding square root of a number is to treat it as the sum of several squares, and repeatedly use Euclid I. 47. For example to find the square root 21

$$21=16+4+1.$$

Make a right-angled triangle of sides 4″ and 2″, at one end of the hypotenuse draw a perpendicular 1″ long. Join the end of this perpendicular to the other end of the hypotenuse.

Then the square on this line $=16+4+1=21$ square inches, therefore the line itself $=\sqrt{21}''=4{\cdot}58$.

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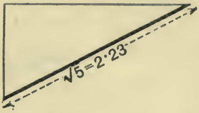

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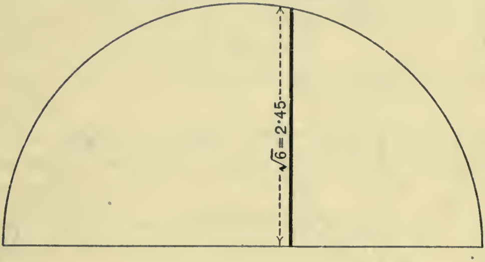

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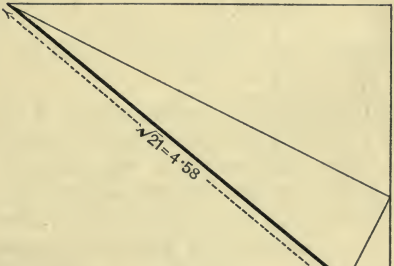

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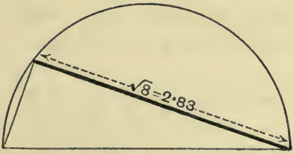

CHAPTER XIV.

CONIC SECTIONS.

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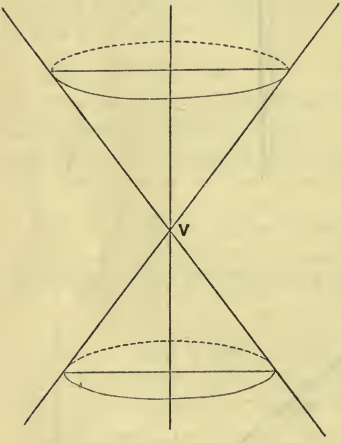

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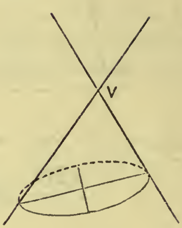

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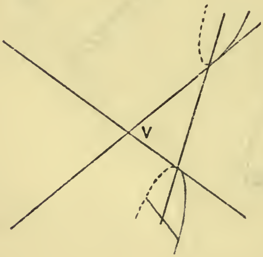

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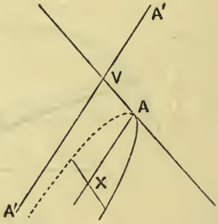

If a right-angled triangle revolve about one of the sides containing the right angle, the solid thus formed is called a right cone. This is only half of a complete cone as treated of in the theory of conic sections, another equal cone is placed with its vertex opposite to that of the first, and both cones are supposed so large as to extend without limit. Another method of describing such a complete double cone is to take a straight line intersecting a given fixed straight line called the axis at a given point V, and to suppose it to revolve round the axis always making a fixed angle with it, and passing through the given point V on the axis.

This cone when cut by a plane gives what is known as a conic section.

If the plane passes through V the section is two straight lines. The section made by a plane cutting the axis at right angles is circular (fig. 1).

An oblique section, not parallel to the edge of the cone, but cutting one part of it only is called an ellipse (fig. 2).

An oblique section cutting both parts of the cone is a hyperbola (fig. 3).

A section parallel to an edge of the cone is a parabola (fig. 4).

It is found that all these sections can be described by a certain law.

Take a fixed point S called the focus, and a fixed straight line MX called the directrix, then if P be a point on the curve, and PM be drawn perpendicular to the directrix the ratio of SP to PM is the same for all points on the curve. If this ratio is less than unity the curve is an ellipse; if equal to unity, a parabola; and if greater than unity, a hyperbola.

The ellipse and hyperbola have each two foci and two directrices, and there is a point midway between the foci called the centre.

The straight line joining the foci is called the major axis, or sometimes more shortly the axis.

In an ellipse the straight line through the centre at right angles to the major axis is called the minor axis. The major axis is the greatest chord through the centre, and the minor axis the least.

(127) To describe an ellipse when the major axis and minor axis are known.

Let ACA′ be the major axis of the ellipse and CB half the minor axis, C being the centre.

In an ellipse it is found that if any point on the curve be joined to the two foci the sum of these distances is constant, and is equal to AA′.

Take centre B, and distance equal to CA the circle must cut the axis AA′ at F_1 and F_2 the two foci.

From centres F_1 and F_2 and distances equal to A1, and A′1, make arcs to intersect one another, the points thus found are on the curve for the sum of their distances from F_1 and F_2 is equal to AA′. Similarly making arcs from F_1 and F_2 as centres at distances respectively equal to A2, A′2; and A3, A′3 we find more points on the curve. 1, 2, 3 are any convenient points on the axis.

(128) To describe an ellipse by means of a trammel, the lengths of the major and minor axis being known.

Let PNM be the straight edge of a piece of paper, or of a card, on which are measured PN = CB and PM = CA. Move the trammel about in any way always keeping M and N upon the minor and major axis, then P traces out the ellipse.

From this construction another is evident. Make circles centre C, and distances respectively CA and CB. Draw any radius CRQ; from R draw a parallel to CA, and from Q a parallel to CB meeting it at P. The point P is on the curve.

By taking different positions of the radius CRQ we can find any number of points on the curve.

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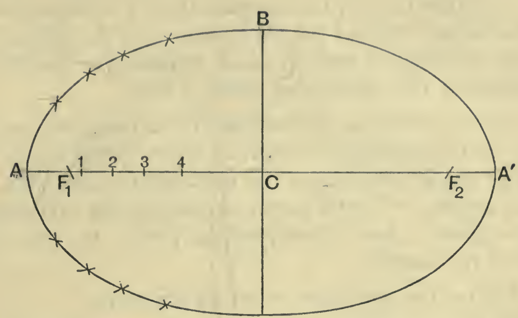

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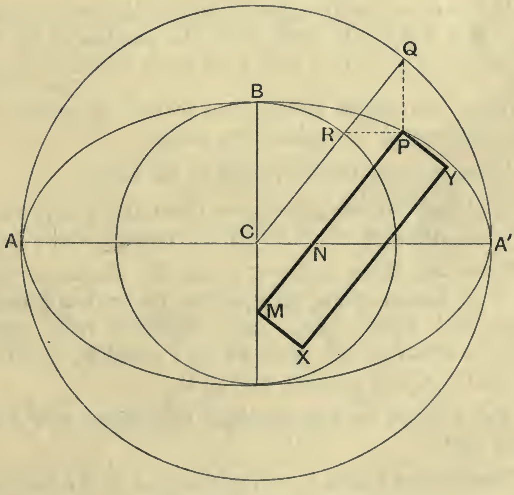

(129) To describe an ellipse within a given parallelogram to touch the sides at their middle points.

Let PQSR be the parallelogram D, G, E, F the middle points of the sides. DE and GF are diameters of the ellipse, and are called conjugates, because each diameter is parallel to the tangents at the extremities of the other.

Divide one of the semi-diameters into any number of equal parts and number the divisions as shewn in the figure. Divide half of one of the sides of the parallelogram adjacent into the same number of equal parts, numbering the divisions in the order given. Join corresponding points to extremities of the other diameter.

The intersections give points on the curve.

If the parallelogram were a square we should obtain a circle, and in this case the proof of the construction is deduced from the fact that the lines intersect at right angles and their intersections lie on a circle.

If this figure (of the square and circle) were drawn on the glass of a window in dark lines, the shadow of the figure on any white plane surface within the room would be figure 129.

(130) An ellipse being given to find its centre, axes, and the tangent and normal at any point.

Draw any two parallel chords in the ellipse.

Find their middle points, join them, this line produced both ways is a diameter which bisected gives the centre, C.

Draw any circle centre C to cut the ellipse in four points, the lines bisecting the arcs of the circle thus found are the major and minor axis. Arcs described from centre B at distance equal to CA meet AA′ in F_1 and F_2, the foci. Join F_1P and F_2P and produce F_1P to L.

The tangent or line touching the ellipse at P bisects the angle F_2PL.

The normal which is at right angles to the tangent at the point of contact bisects the angle F_1PF_2.

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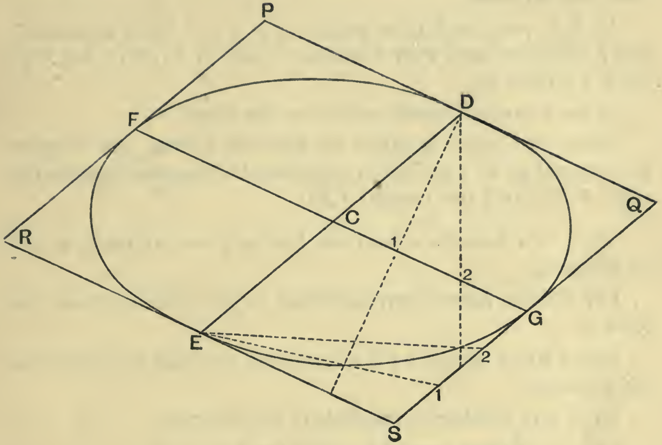

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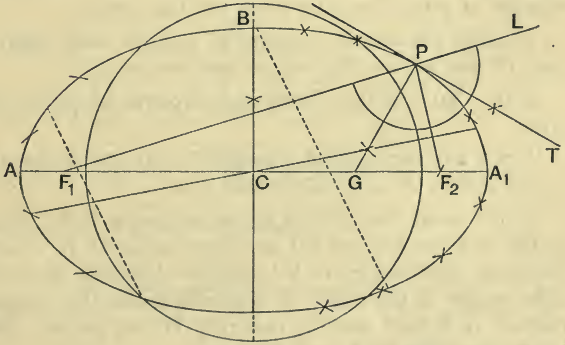


(131) To describe a hyperbola having given its major axis AA′ and its foci F_1 and F_2.

In the hyperbola *the difference* of the distances of any point from the foci = AA′.

In F_1F_2 produced take points 1, 2, 3.... With centres F_1 and F_2 describe arcs with distances equal to A1, A′1; A2, A′2; A3, A′3, and so on.

In each case we obtain points on the curve.

When the curve is given we find the centre, axis tangent and normal as in 130, but in this case the tangent bisects the angle F_1PF_2 and the normal F_2PL.

(132) To describe a parabola having given its focus S and its directrix.

Let SX be drawn perpendicular to the directrix from the point S.

Bisect SX at A, A is on the curve and is called the vertex of the parabola.

Draw any number of parallels to the directrix.

Let one of these meet SA, called the axis, in N.

A circle described with centre S and a distance equal to NX meets the line NP in points on the curve. Similarly any number of points may be found on the curve.

If along the axis we measure ST and SG each equal to SP, then PT and PG are the tangent and normal at P.

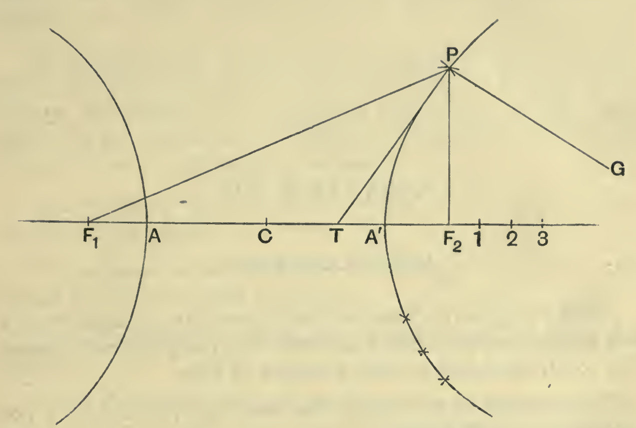
COR. To find the focus and directrix when the curve is given.

Draw any two parallel chords PP′ and QQ′, bisect them at U and V, then UV is parallel to the axis.

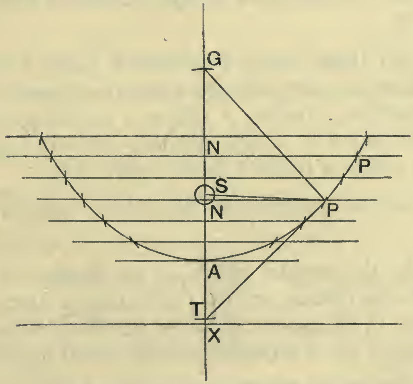
If UV meets the curve in R, the tangent at R, RT, is parallel to PP′ and the normal RG at right angles to it. Draw any chord at right angles to UV, the line bisecting this chord at right angles is the axis. If the axis meets the tangent and normal at R in T and G, bisecting TG we get S. The axis meets the curve in A, and AX = AS.

Draw the directrix at right angles to the axis through X.

131.



132.



CHAPTER XV.

MISCELLANEOUS.

WHEN a figure is contained by circular arcs or other curves that meet at angles pointed inwards, and not produced beyond this point, the figure is said to consist of foils.

The junction of two curves that meet at a point but are not produced is called a cusp.

When the two curves that meet at a cusp touch the same straight line at this point they are said to be tangential.

A foiled figure is described by the number of arcs or lobes that contain it.

A trefoil has three foils; a quatrefoil, four; a cinquefoil, five; when there are more foils than five the figure is called a polyfoil or multifoil. When a figure is contained by curves some of which meet at angles pointing outwards and others inwards it is called a pointed foiled figure (139).

A figure contained by three arcs of circles is called a spherical triangle.

Let ABC be the angular points of an equilateral triangle. With centre A describe an arc BC; with centre B, an arc CA; and with centre C, an arc AB, then the figure contained by the three arcs AB, BC, CA is an equilateral spherical triangle.

Two arcs situated as BC, CA are two sides of an equilateral spherical triangle.

In (133) and (134) it is shewn how to construct in an equilateral triangle and in a square a trefoil and quatrefoil of semicircles.

To inscribe in a circle, or in a regular polygon a foiled figure of semicircles, we divide the circle or polygon into sectors or triangles of the required number by straight lines drawn to the centre, then by (94), or (93) inscribe a semicircle in each sector or triangle so obtained.

An arch may be formed of circular arcs or other curves symmetrically placed.

The supports upon which the extremities of the arch rests (A and B, 135) are called its piers.

The arch is said to spring from A and B, so that AB is the springing line.

C is the crown of the arch, the lower line of the arch-stones is the intrados or soffit ; the upper, the extrados or back.

The arch-stones are called voussoirs, and the highest stone the keystone.

AB is the span of the arch, and the distance of C above AB its height.

The junctions of the arch-stones are normal to the arch.

Spirals may be composed of circular arcs or described by special rules.

The most important are as follows :

(1) The Spiral of semicircles (136).

(2) The Spiral made up of circular arcs on *an eye* or centre.

(3) The Spiral sometimes called the Ionic Volute made by arcs of circles (149).

(4) The Involute of a circle (146).

(5) The Spiral of Archimedes (147) ; and

(6) The Logarithmic Spiral.

The rule for describing this last spiral is to draw any number of straight lines from a fixed point called the centre each making the same angle with the next to it (for example 30°). On one of these lines take a fixed distance from the centre, on the next a distance bearing a fixed ratio to it (for example $\frac{11}{10}$); on the third line a distance $=\frac{11}{10}$ of that on the second; on the fourth, a distance $=\frac{11}{10}$ of that on the third; and so on.

Draw the curve passing through these points.

The curves forming the true Ionic volute approach for most of their length, except near the centre, to the logarithmic spiral.

There seems some doubt whether this spiral was used to construct the curves or whether they were drawn by eye. [See the Penny Cyclopædia under the head Column, and the English Cyclopædia under the heads Column, Volute or Spiral.]

In the Spiral of Archimedes the normal at any point touches a fixed circle, the centre of which is at the centre of the spiral, and its radius is the difference between any two radii of the spiral that contain an angle of $180° \div \pi =$ about $57°\frac{1}{2}$.

In any curve the tangent at any point is drawn at right angles to the normal.

The Spiral of Archimedes is used in mechanical engineering as the shape given to cams to impart linear motion to a point, when the cams have uniform circular motion. [See the Heart shaped Cam in books on Engineering Drawing and Design.]

Ovals made by arcs of circles, like spirals, when changing in construction from one arc to another must have the point of junction on the line joining the centres of the circles. An example is given in (141).

Cycloids are described (142) to (145); and construction of figures from data (150) to (155); and other miscellaneous examples (156) to (162).

Verbal instructions are here given without figures for the construction of some interesting problems.

(*a*) To describe an ellipse when the foci and major axis are given by means of a piece of thread. Tie the ends of a piece of thread, or fine string to pins fixed upright in the paper at the foci. The thread must be of such a length that when stretched by the point of a pencil it just reaches both ends of the major axis.

Move the point of a pencil so as to keep the string stretched but not too tight. It will trace out the ellipse on the paper.

(*b*) Two men are standing at points A and B on the same side of a straight stream CD.

Find a point P in CD so that the men may meet at P and that the sum of the distances they run AP and BP may be the least possible.

Draw BNQ perpendicular to CD meeting it in N and produce $NQ = BN$.

Then join AQ; it cuts CD in the required point P, and AP, BP are equally inclined to CD.

(*c*) On a map, scale 1 mile to 1 inch, four points ABCD are observed as angular points of a square on a side of two miles. The part of the square BAC forms a promontory, the point D being inland.

On a ship at sea, at X, the angles $BXA = 60°$, and $CXA = 30°$ are observed.

Find the position of X, and the angle DXA.

On BA describe a segment of a circle, on the side towards the sea, containing an angle 60°; and on CA, as segment containing 30° [32].

These segments intersect in X. Measure DX, join XA and DX, and measure the angle DXA.

This problem is called the "three station problem" because by observing the angles made by joining an unknown point X to three known stations A, B, C, we find the position of X.

(*d*) Given three circles, two of which are equal, and their centres equally distant from that of a third circle, to find a circle that is touched by all of them internally.

Let A be the centre of the third circle, equally distant from B and C the centres of the other two. Draw AD perpendicular to BC and let DA meet the circle, centre A, in M.

Measure ML along MD a distance equal to the radius of one of the other circles. Join BL and bisect it at right angles by a straight line meeting MD in O. O is the centre of the circle, and its radius is OM.

To find the circle touching the three circles externally, take M the point where AD (not produced), cuts the circle centre A, and ML is measured along MA, away from the point D.

OM is the radius of the circle required, and O its centre, which is found as before by bisecting BL at right angles.

A similar method is given when the three circles are unequal but it is only approximately true. Another more difficult solution is given in notes to examples.

(*e*) On Arithmetic, Geometric and Harmonic Means.

The Arithmetic Mean between two straight lines is half the sum of their lengths.

It is useful to note that if AB and CD be two parallel unequal straight lines joined towards the same parts by AC and BD and if AC and BD be bisected in P and Q, then PQ is parallel to AB or CD and is equal to half their sum, and is therefore their Arithmetic Mean.

The Geometric Mean between two straight lines is the mean proportional between them. This is true also for numerals.

The Harmonic Mean between two straight lines (or between two numerals) is the third proportional to their Arithmetic and Geometric means.

(*f*) To describe an equilateral triangle equal in area to any triangle ABC.

An easy solution of this problem will be found (122). This method is given as an example of one of the uses of mean proportional.

Bisect a side of the triangle BC, at D, by a straight line ED at right angles to BC.

Let a parallel to BC through A, meet ED in E. If we join EB and EC (but this is not necessary) we get an isosceles triangle equal in area to ABC.

Draw an equilateral triangle HBC on the base BC.

The point H will lie on ED. Find a mean proportional DK to HD and ED.

DK is the altitude of the equilateral triangle required, which is completed by drawing parallels to HB and HC.

(*g*) To bisect a triangle ABC by a straight line perpendicular to its base BC.

Bisect BC at D. Draw AE at right angles to BC.

Find a point K in BC such that BK is a mean proportional between BD and BE. Draw a perpendicular to BC through K, this bisects the triangle.

(133) To inscribe in an equilateral triangle a trefoil of semicircular arcs.

Bisect AC and AB in E and F. Make a semicircle on EF, centre X. Draw PX at right angles to AB meeting the semicircle in P. Join P to the intersection of BE and CF.

This line cuts AB where one of the semicircles required, touches it. The centre Y is found by drawing a parallel through this point to PX, and its diameter by drawing KL through Y parallel to FE.

The centres of the other two semicircles are on BE and CF at equal distances from the centre of the triangle.

It will be observed that the semicircles are described by the theory of similar figures, the semicircles on FE and KL being similar and the centre of the triangle their "centre of similitude."

(134) To describe a quatrefoil of semicircles in a square.

Bisect DE a side of the square in A, and bisect AE in B. Make DC = AB.

Join BC meeting a parallel through A to DC in X. Draw XY perpendicular to the diagonal through D.

Y is a centre of one of the semicircles required, the other centres are at an equal distance from the centre of the square, upon the diagonals.

(135) To construct an equilateral arch, on a span AB.

With centre A and distance AB describe an arc, and centre B and distance BA.

Let these arcs meet in C. The outer arcs have the same centres A and B.

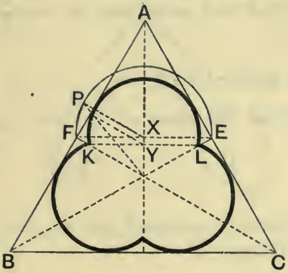
The junctions of the stones forming the arch are indicated by straight lines through A and B as shewn in the figure, they are said to be normal to the arch because they are at right angles to the tangent to the arch where they meet it.

(136) To describe a spiral of semicircles.

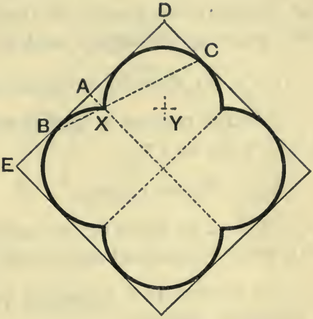
Take a fixed straight line AB produced both ways. On it describe a semicircle centre O.

With centre A and distance AB describe a semicircle BC. With centre O and distance C describe a semicircle, and so on, taking centres alternately at A and O.

133.



134.



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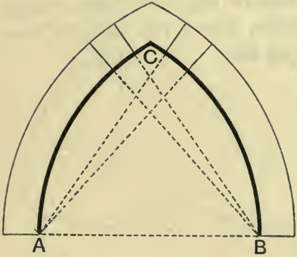

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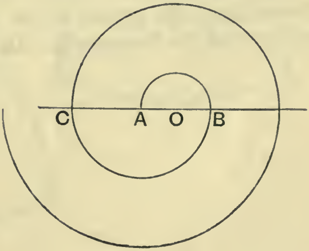


(137) To describe a lancet or pointed arch.

Take the centres A and B outside the span of the arch. With centre A and distance the further end of the span make an arc, and with centre B make a similar equal arc.

The outer arcs have the same centres A and B. As in 135, the junctions of stones of the arch are indicated by straight lines passing through A and B.

(138) To describe an obtuse pointed arch.

Take the centres A and B within the span of the arch and describe as in 137.

(139) Foiled figures.

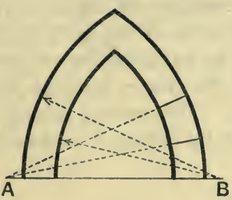
The first of these figures represents a pointed foil described with arcs of spherical equilateral triangles.

The second is a trefoil of tangential arcs described within a spherical triangle.

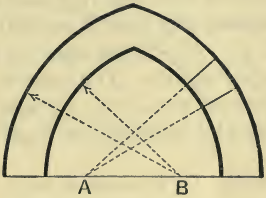
When the spherical triangle is given the point X, centre of one of the arcs of the trefoil is found by bisecting the arc BC in D, drawing the tangent PD at right angles to BD to meet the straight line joining the centre of the figure to A. Bisect the angle DPA by XP meeting BD in X.

The centres of the other two arcs of the trefoil are equally distant from the centre of the figure, along the straight lines joining it to the bisections of AB and BC.

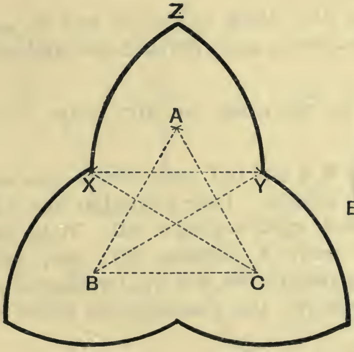
137.



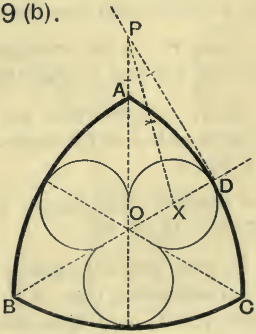
138.



139 (a).



139 (b).



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(140) To describe an elliptical arch.

Let AA′ be the span of the arch, and S and H the foci of an elliptical arch described on the span AA′.

Draw the ellipse on the line AA′ as major axis and foci S and H by one of the methods already given (127 to 130).

Divide the arch up into any number of equal divisions measuring first from A towards the crown of the arch, and then from A′. We cannot take the arcs as equal but must measure by equal chords. The junctions of the stones must be normal to the arch as in (135, 137 and 138). The normal at any point is found by bisecting the angle made by joining it to S and H.

(141) To describe an oval.

Take a straight line ABCDE, bisected at C, and let CB = CD.

In this case CB is also made equal to BA. In other words AE is divided into four equal parts.

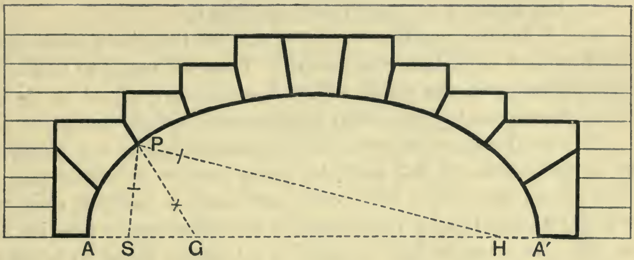
Take H on the straight line bisecting BD at right angles. Join AH, EH and produce them.

Make a semicircle on BD. With centres A and E, and distances AD and EB make arcs to meet AH and EH produced in K and L.

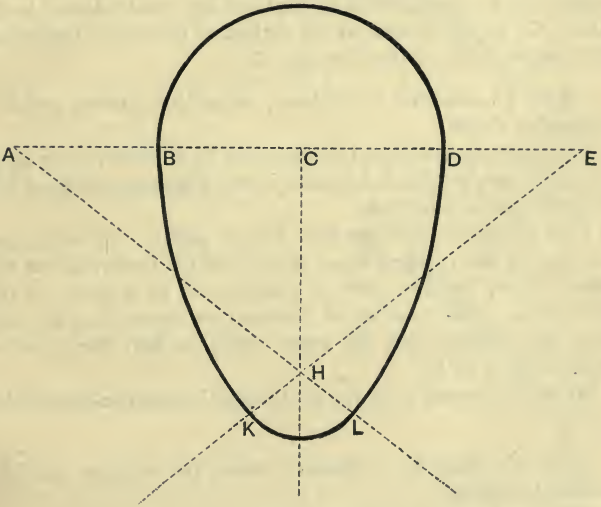
Complete the figure by describing an arc centre H at distance HK or HL.

In some ovals the points B, A and D, E are taken to coincide. The construction then is as follows. Take a straight line AD, bisect it at C, and take CN at right angles to AD. Make the semicircle on AD. With centre A, distance AD; and with centre D, distance DA make the arcs BK and DL, terminated on straight lines drawn through H: and complete the figure as before.

140.



141.



(142) To describe a cycloid.

If a circle is made to roll along a fixed straight line, any fixed point on the circle describes a cycloid.

In the figure half the cycloid is shewn.

Let A be the position of the centre of the circle when the fixed point P is at its highest point. Take BC equal to half the circumference of the circle which is very nearly equal to three times the radius together with one-seventh part of the radius.

Divide the circumference of the semicircle into any number of equal parts and BC into the same number of equal parts. Now when the circle has come into such a position that its vertical radius is at XY, the point P has come into a position Q so that the triangle QXY is equal to a triangle found by joining A, B to a corresponding point on the first circle. This point is found by measuring from P along the circle the same number of divisions as there are in BY. Therefore Q can be found by making arcs from X and Y as centres with distances equal to the sides of the corresponding triangle on AB. Any number of points can be found in this way and the curve drawn in by hand. QY is the normal to the cycloid at Q and the tangent at right angles to the normal through Q.

(143) To describe a trochoid, when the moving point is within the circle.

A trochoid is described as a cycloid by a circle rolling upon a straight line, but the fixed point is taken within, not upon the circumference of the circle.

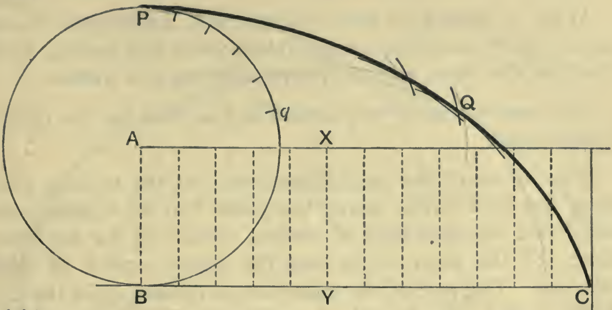
It is the path described by a bicycle pedal. As in the case of a cycloid BC is taken equal to half the circumference of the rolling circle, but the point Q corresponds to a point on the inner circle. The number of divisions are counted on BY, and along the divisions on the inner circle to find the position corresponding to Q.

QY is the normal at Q, and the tangent is taken perpendicular to it.

(144) To describe a trochoid when the moving point is outside the circle.

The construction is similar to 143.

142.



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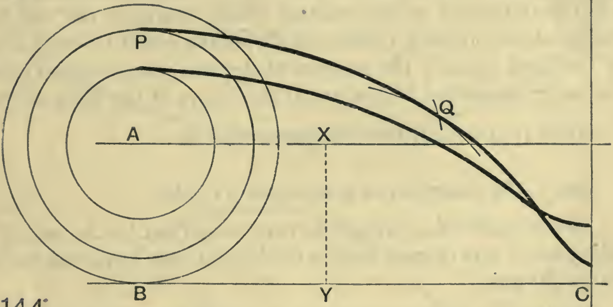

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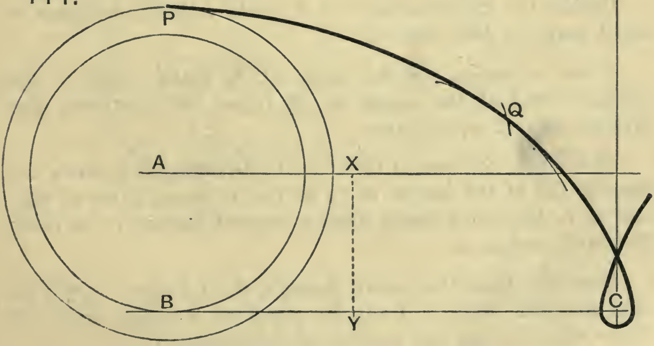


(145) To describe the epicycloid and hypocycloid.

When a circle rolls upon a fixed circle, a fixed point on the moving circle describes an epicycloid when the moving circle is outside the other, and a hypocycloid when it is within.

The construction is very similar to that used for the cycloid in both cases.

First measure the semicircumference of the moving circle along the fixed circle, taking the point P at its highest point, and divide the semicircle of moving circle, and the arc (found above) of the fixed circle into the same number of equal divisions. The rest of the construction follows as in the case of the cycloid. To find the arc of the fixed circle equal to the semicircumference of the moving circle, multiply 180° by the radius of the moving circle, and divide the result by the radius of the fixed circle. The number of degrees thus obtained gives the angle subtended by the arc at the centre of the fixed circle.

As in (142 to 144) QY is the normal at Q.

(146) To describe the involute of a circle.

The involute of a circle is a curve traced out by the end of a string when unwrapped from a circle, and kept stretched out in a straight line.

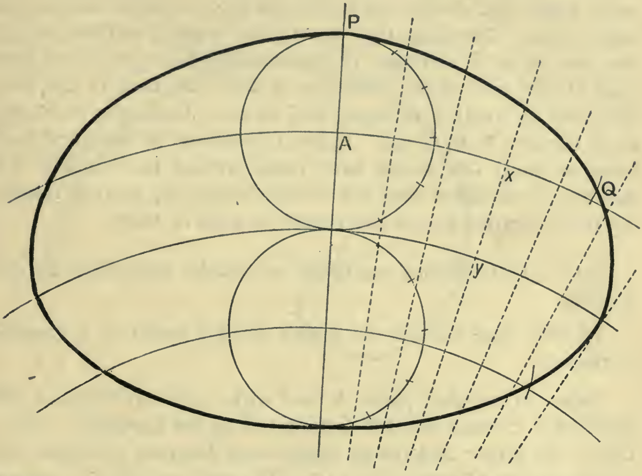
Divide the circumference of a circle into any number of equal parts, in this case 12.

Draw a tangent to the circle at A, made equal to the circumference of the circle, or $3\frac{1}{7}$ times the diameter, also divided into 12 equal parts.

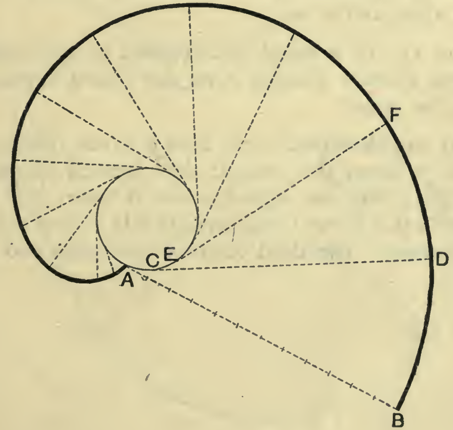
At the next division of the circle to A namely at C, draw the tangent CD of the length of 11 of the 12 equal parts of AB; and at E the next division draw a tangent having 10 of these divisions, and so on.

Draw (by hand) the curve through, B, D, F, &c. It will be seen that the curve is drawn by wrapping a string upon the circle which is just the reverse of unwrapping it.

145.



146.



(147) To construct the spiral of Archimedes.

Divide the circumference of a circle into any number of equal parts, and divide the radius OA into the same number of equal parts. OB along the next radius in order to OA through the next point of division of the circumference, is taken less than OA by one of the divisions of OA; OC next to OB, less than OB by another division, and so on. Draw the curve by hand through A, B, C, &c. If two revolutions of the spiral had been required OA would have been divided into double the number of divisions that the circumference is, so that taking the radii in order we get two points on each of them.

(148) To construct any spiral of circular arcs when its *eye* is given.

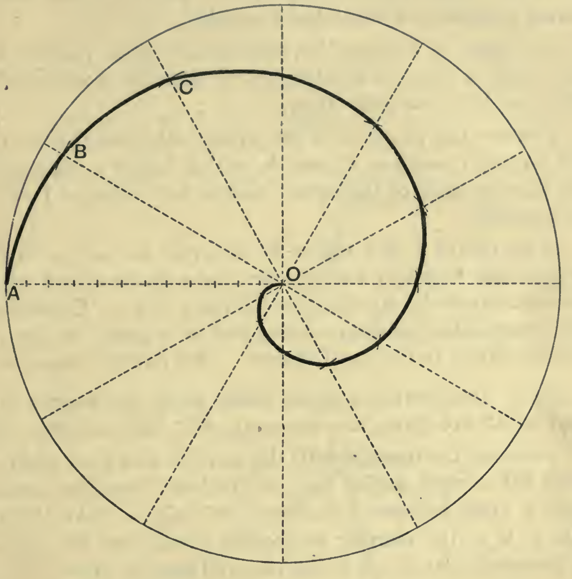
In this case the *eye* or centre of the spiral is a regular pentagon.

Take an angular point A and with centre A distance AB describe a circular arc BC terminated by DA produced. Next take D as centre and DC as radius and describe a circular arc terminated by FD produced to E. The next arc has F as centre and FE as radius, and so on.

When an eye of a spiral is composed of any number of given centres whether forming a regular closed figure or not, the rule is the same.

The first arc described must have a given radius, and is drawn until it meets the straight line through the first and second centres; then the second centre is taken, and the arc continued with this centre to the straight line joining the second and third centres. The third centre is now taken and the rule repeated.

147.



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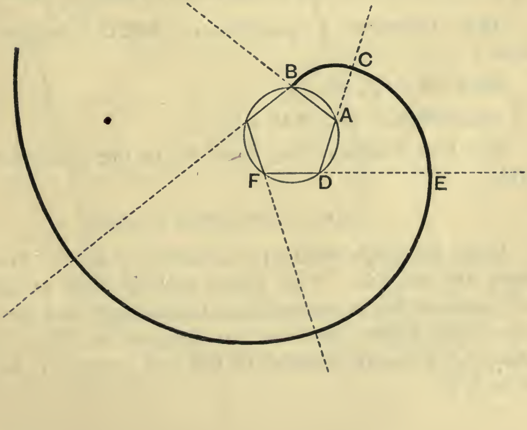


(149) To construct the spiral known as the Ionic volute.

In the figure the eye only of this spiral is given, which is drawn as explained in the last example.

A square is bisected by two straight lines parallel to its sides each of which is divided into six equal parts and numbered from 1 to 12 in the order shewn.

Produce the diagonal of the square that bisects the straight line joining 1 and 2 to a point A, so that A is at a distance from the angular point of the square nearest to it equal to four times its diagonal.

With centre 1 and radius A1 describe an arc to meet the straight line joining 1 and 2. Next take centre 2, and continue the arc to meet the straight line joining 2 and 3. Continuing in this manner the volute is completed at a point on the circle circumscribing the original square. This circle is drawn in.

(150) To describe a figure ABCD when the lengths of AB, BD, and AD are given, also the angle ABC, and that BC = CD.

Construct the triangle ABD the sides of which are given, and bisect BD at right angles by a dotted line: then the remaining angle C must be upon this line if BC = CD. Make the angle ABC at B of the number of degrees given, then BC will meet the perpendicular to BD at the required angular point.

(151) Construct a quadrilateral ABCD from the following data.

Sides AB = $1\frac{1}{2}''$, BC = $1\frac{1}{2}''$.

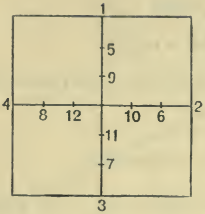
Angles ABC = 105°, BAD = 75°.

The four angles of the figure lie in the circumference of a circle.

(*Art Examination Papers.*)

Make the angle ABC = 105°, that is 45° + 60°. Measure $1\frac{1}{2}''$ along AB and BC. If we bisect each of them at right angles and produce the bisecting lines to meet we find the centre of the circle, which describe at distance A, B or C. Since BAD = 75°, draw AD parallel to BC and produce it to meet the circle in D.

149.



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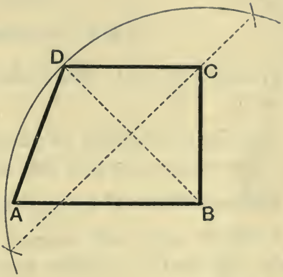

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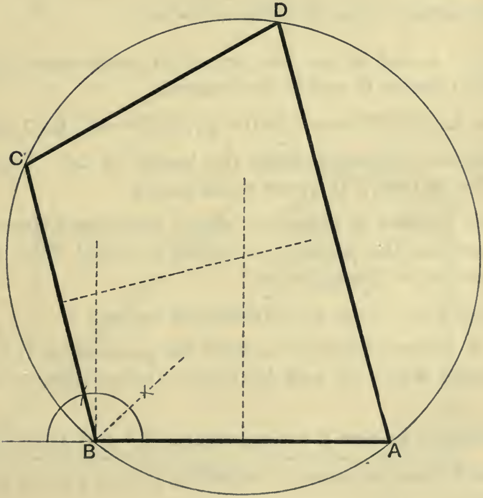


(152) Make an irregular pentagon ABCDE from the following data:

Sides AB = $2\frac{1}{8}''$, BC = $1\frac{1}{4}''$, CD = $2\frac{3}{8}''$, DE = 1″.

Angles ABC = 105°, ABE = 30°, BAE = 105°.

Then make a similar figure in which the side corresponding to AB = $1\frac{5}{8}''$.

(*Art Examination Papers.*)

Measure AB = $2\frac{1}{8}''$ and make the angles 105° at A and B and 30° at B.

The lines indicated as AE and BE making 105° at A and 30° at B intersect in E one of the angular points of the figure.

Measure BC = $1\frac{1}{4}''$ along the line BC.

Describe arcs with centres C and E and distances respectively $2\frac{3}{8}''$ and 1″, these arcs intersect in D the remaining angular point.

The similar figure is made as in (100).

(153) A and B are two points 40 yards apart, from which two other points C and D are observed.

The angle ABC = 90°, BAC = 30°, ABD = 60°, BAD = 45°.

Measure and write down the length of CD. [Scale which need not be drawn 16 yards to the inch.]

This method of observing angles from two known points in order to find the position of others is called "the method of intersection or triangulation."

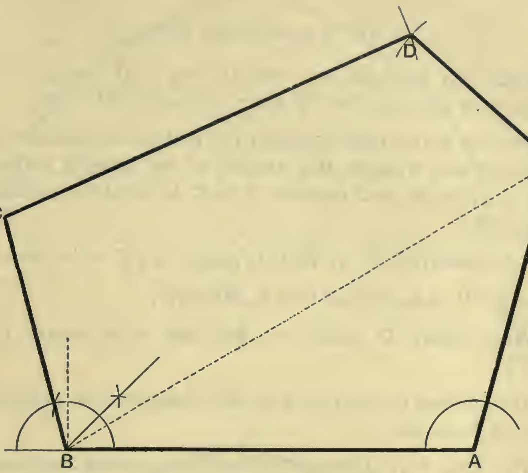
Draw AB = $2\frac{1}{2}''$ or 40 sixteenths of an inch.

It is evident that we can find the position of C by making the angles BAC = 30° and ABC = 90°, and producing the lines to meet.

Similarly we find D because BAD = 45° and ABD = 60°.

Join CD and measure it we find it about $\frac{15}{16}$ of an inch which represents 15 yds.

152.



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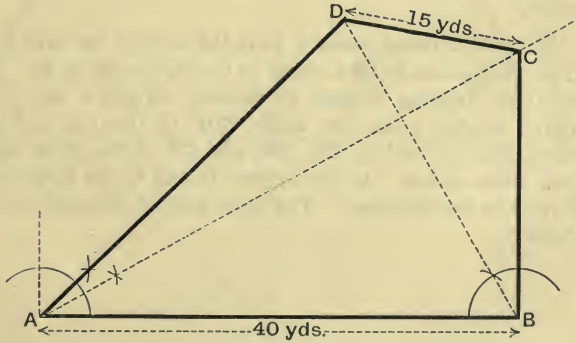


(154) Draw the four-centred arch, from the dimensions given.

(*Art Examination Papers.*)

Draw the straight line AB, 3″ long and bisect it at right angles by a straight line $1\frac{1}{2}''$ long. Measure BD = $1\frac{1}{4}''$.

We find C from the figure by the method of similar triangles. Construct the triangle B*ca* exactly of the lengths given in the diagram, join B*c* and produce it to C to meet the parallel to *ac* through A.

On measuring CF we find its length is $2\frac{1}{2}''$ as required.

Join CD and produce it to E, DE = $1\frac{1}{4}''$.

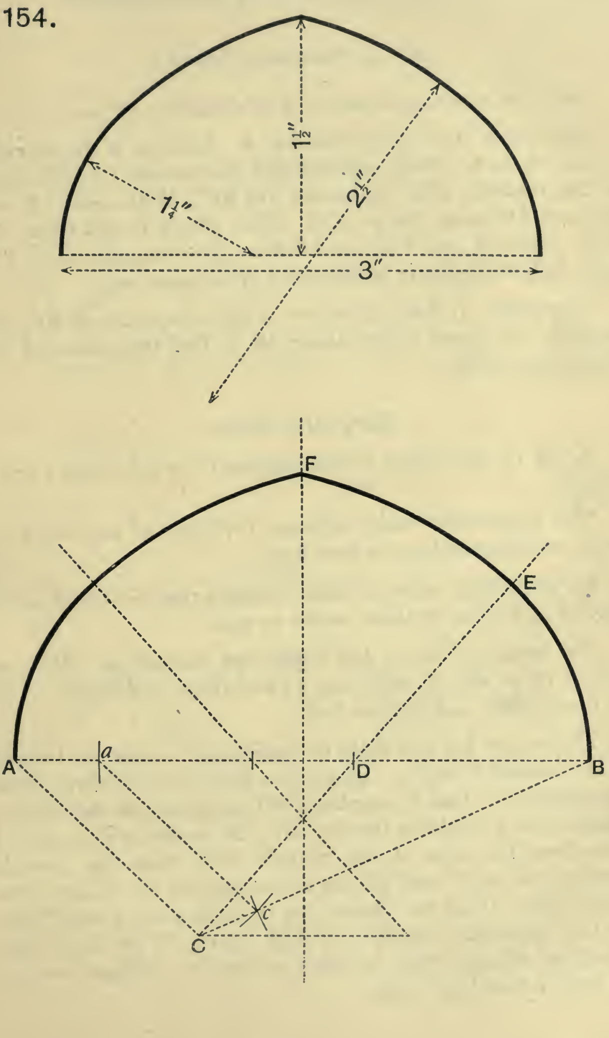
With centre D make arc BE, and with centre C make arc FE.

The centres C and D are on the straight line through E the point of junction.

The other half of the arc is found by drawing a perpendicular from C to the bisecting line of the figure, and producing it to an equal distance the other side, and by measuring a distance from A along AB = DB. These give the centres for the arcs on the other side, which are described in the same way as BE and EF.

Usually it is found easier to have the radii of the arcs given and the inclination of CDE either to the vertical or to AB. The method of drawing is then as follows. Measure BD of the required length, make the angle BDE of the required size. Produce ED to C making DE = DB, and CE of the given length of the other radius. As the centres D and C are now known the arcs can be described. The other half of the arch is found as before.

154.



(155) Describe the figure to the given dimensions.

(Art Examination Papers.)

Describe a rectangle the sides of which are $3\frac{1}{2}''$ and $1\frac{1}{4}''$.

Bisect one of the longer sides in A. Join one of the opposite corners B, to A. With centre A and distance AE (the bisection of the opposite side) make the arc EC. With centre B and distance BC make the arc CF. With centre B make arc GD. With centre A, and distance AD draw the remaining arc. The other half of the figure is described in the same way.

Bisect BF, to find the radius of the semicircle on BF, and measure the same radius along AE to find the centre of the remaining circle.

Marquoise Scales.

A box of Marquoise Scales consists of two flat rulers and a triangle.

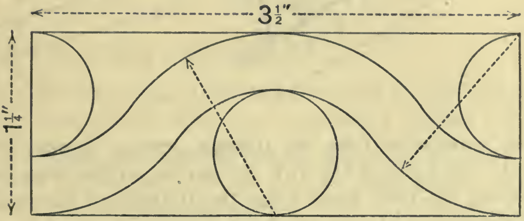
The rulers shew many different divisions of an inch from 10ths to 60ths and are a foot long.

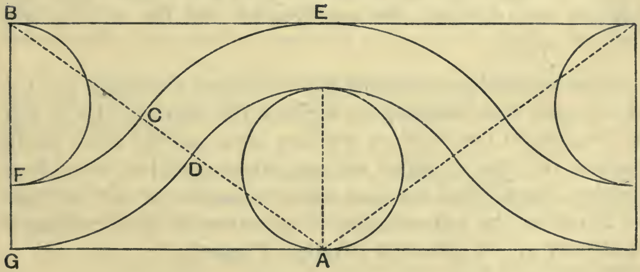
On examining some of these divisions they are found not to refer to inches but to three inches as unit.

For instance look at the upper line marked 40. Measure from 0 to 40 with an inch rule, we find these 40 divisions make up three inches and not one inch.

The reason for this scale of measuring is found by looking at the wooden triangle. Its longest side is exactly three times the shortest, so that if we place the longest side of the triangle against this 40 scale on the flat ruler, the arrow on zero, draw a line along the side of the triangle, then slide the triangle keeping the ruler fixed till the arrow rests on the 40, and draw another line along the side of the triangle, these parallel lines on the paper are exactly one inch apart. If we draw lines when the triangle rests at each of the 40 divisions we get 40 lines parallel $\frac{1}{40}''$ apart.

155.





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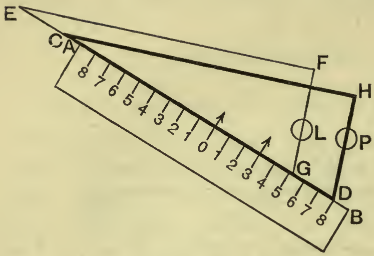

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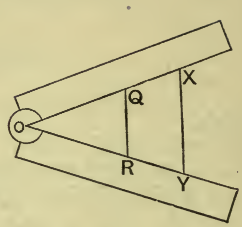


As an illustration take the triangle resting against a flat ruler as in the figure **EFG** with the arrow on the zero point. Now keeping the ruler fixed move the triangle to the position **CHD** till the arrow rests on the third division.

The straight lines drawn along the edges **EF**, **CH** are parallel and their distance apart is equal to one division on the ruler, or one-third of the distance the arrow on the triangle has been moved.

If the edge **HD** were half of **CD** then if the arrow is moved through two divisions the parallels **EF** and **CH** will be one division apart.

This is useful to remember for if we have a triangle 30°, 60° 90°, as used with ordinary set squares, the shortest edge is half the longest, so that with an ordinary inch rule (an arrow being marked on the triangle) we can draw parallels any given distance apart. For instance with a triangle 30°, 60°, 90° and an arrow on the longest side, if we move the arrow along a fixed ruler $1\frac{1}{2}''$ we can draw parallels $\frac{3}{4}''$ apart.

Another use of set squares or of Marquoise scales is shewn in the above figure (1).

Place the edge **EF** along any given straight line, and slide it along a fixed ruler till the edge **HD** passes through **P** then **HD** is a perpendicular through **P** to **EF**.

Similarly if the triangle rested on CH, the ruler AB being fixed, we can move the triangle till the line FG passes through L a given point, then FG is a perpendicular to CH through L. This is the method used also with set squares to draw perpendiculars through given points to given straight lines.

The Sector.

This instrument is a jointed ruler.

Of the lines on the ruler those marked L and Pol. are most used; these are called the line of lines and the line of polygons.

The figure (2) shews the use of the line of lines or L. Open the ruler hinged at O till the line XY measures a certain given distance from the division marked 10 to 10 on the opposite side. If then we take QR from the division 6 to 6 on the opposite side, XY being kept the same, $QR = \frac{6}{10}$ or $\frac{3}{5}$ of XY.

Similarly if XY were measured in the first place from 7 to 7 and QR from 5 to 5 keeping XY the same, then $QR = \frac{5}{7}$ of XY.

We can also find lines in greater proportion to one another for if we opened the sector till QR measures a given distance from 5 to 5, and then keeping the sector in the same position measure the distance XY from 7 to 7 then $XY = \frac{7}{5}QR$.

To describe any regular polygon say of seven sides, open the sector till on the lines marked Pol. from 7 to 7 we measure the base required.

Then without moving the sector measure from 6 to 6, this length will be the radius of the circle circumscribing the polygon. Describe the circle and step off the side round it.

Or if the radius of the circumscribing circle be given, open the sector till the distance from 6 to 6 is the radius of the circle, then measure the side of the polygon required, from 7 to 7 for a heptagon, 8 to 8 for an octagon, and so on.

The Protractor.

In the Introduction will be found a figure shewing the use of the protractor to measure angles up to 180° : in the Chapter on Scales, the use of the diagonal scale: and (111), (112) shew how to obtain an angle from the scale of chords. On some protractors angles are given up to 360°.

Draw a vertical straight line **SON**, the upper end being marked **N**, the lower **S**, and **O** being about midway between **N** and **S**.

If the edge of the protractor be laid along **NS** with the arrow at **O**, the protractor being to the right of **NS**, the angles measure from **N** round to **S** up to 180°. Now lay the protractor on the left of the line **NS**, the arrow at **O** and its edge along **NS**, on this side use the scale from 180° to 360°. The reason for doing so is this, in using the compass to observe angles we take a line drawn to the north (**N**) and measure angles eastwards right round to the south (**S**) and still in the same direction up to north again. It is necessary to use angles from 180° to 360° to distinguish west from east.

On Resection.

In figure 153 the method of intersection was explained, that is, given two known points to find another by observing angles from the known points.

In resection or "interpolation" we observe two known points from the unknown point to find its position. The rule is as follows :—draw the line **NOS** (as just explained), **A** and **B** being the known points, and draw at **A** and **B** angles the reverse of those observed at **C** the unknown point. The lines so obtained intersect at **C**.

For example suppose we take any two points **A** and **B** as known and state that at **C** the angle 240° with the north is observed for **A**, and 150° for **B**. Draw through **A** and **B** parallels to **NOS**. Measure at **A** the angle 60° with the straight line parallel to **NOS**, or which is the same thing 240° *on the other side* i.e. to the right.

At **B** measure the angle 330° with the straight line parallel to **NOS** or 150° on the other side, i.e. to the left. **C** is the intersection of the straight lines thus obtained.

The Time Scale.

A column of troops marches at the rate of $2\frac{3}{4}$ miles per hour. Construct a scale of time to measure intervals of 10 minutes. Scale 10 miles to 1 inch (Army Exams.).

Find the distance on the map represented by $2\frac{3}{4}$ miles by rules given in Chapter on Scales.

In this case $\frac{11}{40}$″. You can take $\frac{11}{40}$″ from the marquoise scales. Draw a straight line containing 20 of these divisions. Divide the last division on the left into six equal parts. As in plain scales, take the zero point on the right of this division and figure to the right, 0, 1, 2, 3,... *hours* on each division, and *to the left* 0, 20, 40, 60 *minutes* on alternate divisions.

Measurements are taken as from an ordinary plain scale. For example, if two points are given on a road a certain distance apart and the question be asked, How long will it take troops to get from one point to the other? Measure the distance from the map upon the scale. If the points of the compass extend from 5 hours on the right of the scale to 30 minutes on the left, the answer would be 5 hours and a half.

To divide a straight line harmonically.

If four straight lines meet at a point, namely OA, OB, OC, OD, they are said to form a pencil, and if a straight line cuts the four lines in the points ABCD it is said to be a transversal. However this transversal may be drawn the ratio of the rectangle AB.CD to AD.BC is the same.

This property is known as Anharmonic ratio.

When the straight lines OA, OB, OC, OD are so situated that the rectangle AB . CD = the rectangle AD . BC then the ratio is said to be harmonic.

If OB and OD bisect the angle AOC internally and externally then the pencil is Harmonic.

A, B, C are three given points in a straight line, find a point D so that the rectangle AB . CD = AD . BC. (Army Exam. Paper.)

This question is solved as follows :—Describe a circle on AC as diameter. Bisect one of the semicircles, join the point thus obtained to B, and produce it to meet the circle again in O.

Draw OD at right angles to OB to meet AC produced in D. It will be found that OB and OC are the internal and external bisectors of the angle AOC. Therefore D is the point required.

The point D could have been found by describing any circle on AC, bisecting one segment, joining the point to B, producing it to meet the other segment of the circle in O, and drawing OD at right angles to OB.

If A, C, and D were given in the above question to find B, the construction is very similar.

Draw a semicircle on AC, join the bisection of one of the semicircles to D cutting the circle in O.

Draw OB at right angles to OD.

(156) To find the position of the straight line bisecting the angle between two straight lines that do not meet by means of the marquoise scales.

Draw parallels to one of the straight lines PF, QE, RD, and another set of straight lines parallel to the other, namely PA, QB, RC. These sets of straight lines intersecting as indicated in P, Q, R.

The straight line joining P, Q, R is the one required.

It is not necessary to draw more than two straight lines as QB and QE; RC and RD parallel to each, but QB and RC must be at the same distances from one straight line that QE and RD are from the other.

(157) To divide a triangle into any number of equal parts by straight lines parallel to the base.

Let ABC be the triangle and let it be required to divide it into any number of equal parts by straight lines parallel to BC, in this case into four parts.

On the side AC make a semicircle AGHKC.

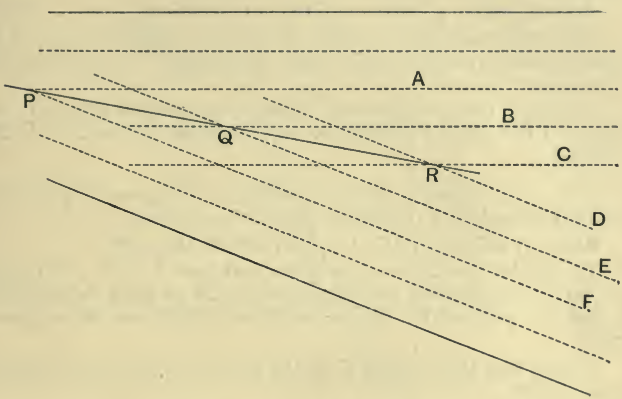
Divide AC into four equal parts and draw DG, EH, FK perpendicular to AC, through the points of division. With centre A and at distances AG, AH, AK make arcs to meet AC, and draw parallels to BC through the points where these arcs meet it.

If a triangle ABM were described upon AB, and parallels to BM drawn through the points at which the parallels to BC meet AB, the figure ACBM would be divided into four equal parts by these two sets of parallels.

The method used in 157 should be compared with 116 and 119 for it depends upon the same proposition of Euclid, namely— If three straight lines be proportional any figure described on the first has to a similar figure described on the second the same ratio that the first straight line has to the third.

Comparing fig. 157, AC, AK, AF are three straight lines in proportion, therefore the whole triangle ABC has the same ratio to the similar triangle described upon a straight line equal to AK that AC has to AF, and AF has been made $\frac{3}{4}$ of AC. In the same way the other parallels to BC cut off $\frac{1}{4}$, and $\frac{1}{2}$ of the triangle ABC and divide it into 4 equal parts.

156.



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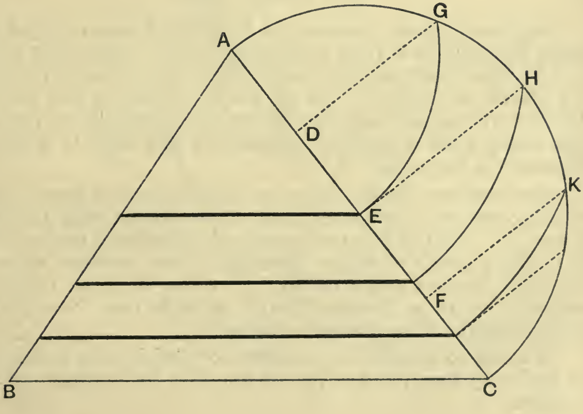


(158) To bisect a quadrilateral ABCD by a straight line through A.

AB = 2″, BD = 3″, AD = $1\frac{1}{2}$″, BC = 1″, CD = $2\frac{1}{2}$″.

Draw the straight line BD = 3″ and on opposite sides of it describe the triangles ABD, CBD the lengths of the sides of which are known.

To bisect the quadrilateral, through the middle point of one diagonal BD draw a parallel to the other AC to meet CD in E. Join AE.

(159) To divide a triangle ABC into three equal parts by straight lines drawn through a point P in one side.

AB = 3″, BC = $2\frac{1}{2}$″, AC = 2″, take AP in AB = $1\frac{3}{8}$″.

Divide AB into three equal parts and join C to P. Through the points of division draw parallels to CP to meet the sides in X and Y. PX and PY divide the triangle ABC into three equal parts.

To see why this is join C to the points of division, denoted by L and M.

Then the triangle CXL = PXL are equal, being on the same base XL and between the same parallels, add to each AXL, then ACL = AXP. Now ACL is $\frac{1}{3}$ of ABC because the base AL = $\frac{1}{3}$ AB.

Similarly BCM = BPY = $\frac{1}{3}$ ABC.

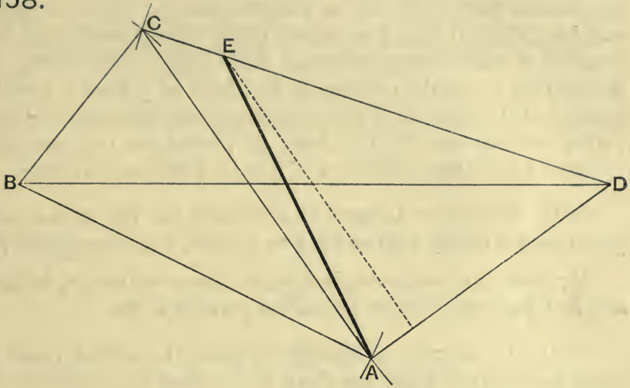
The Vernier Scale.

Suppose a given distance to be divided into 10 equal parts along the edge of a ruler or card, and 11 of these to be divided into 10 equal parts along another edge, then if these two edges are placed in contact it will be found by moving the points of division we can find any part of the given distance to 100ths by measuring from a point of division on one edge to a point of division on the other.

For instance divide 5 inches into 50 equal parts, take a length equal to 11 of these divisions on another edge and divide it into 10. By placing two points of division of the two edges in contact, and the edges opposite to one another along the same line, we can add 4 of the first set of divisions to 5 of the second, that is 40 hundredths of an inch from one scale to 55 hundredths from the other = ·95″.

We can so choose the numbers of division on either scale to make up any hundredths of an inch by addition or subtraction.

158.



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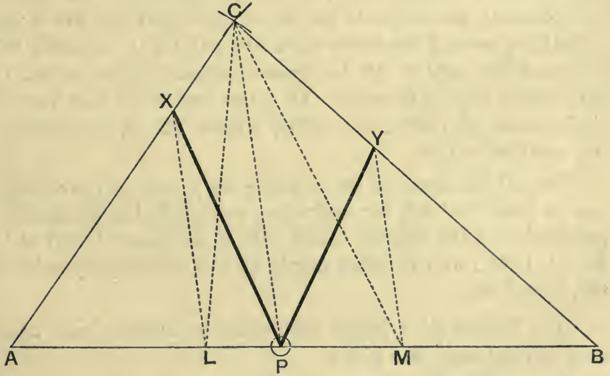


(160) In the example given, one straight line is divided into 10 equal parts each representing 4 yards to a certain scale, and the second edge contains a length equal to three of these divisions divided into four equal parts, that is each division contains a distance representing three yards. By adding or subtracting a certain number of divisions of 3 from a certain number of divisions of 4 we can measure any distance to single yards, for instance by the scale as placed we can measure 17 yards by adding 2 divisions of four to 3 divisions of three.

(161) To draw a tangent to a circular arc BC or to a very flat large arc nearly approaching to a circle, at a given point A.

Measure any two convenient equal distances from A, AB and AC, and draw the tangent through A parallel to BC.

(162) To describe a triangle (1) when the altitude, sum of sides, and vertical angle are given or (2) when the altitude, sum of sides, and base are given.

Both the above constructions depend upon the properties of the inscribed and escribed circle of a triangle.

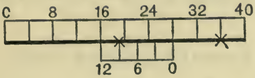
Let ABC be a triangle, X and Y the centres of the inscribed and escribed circles, and XE, YD perpendicular to AB, then the following facts should be remembered.—First AD is equal to half the sum of the three sides AB, BC, CA. Secondly DE = the base BC, and if AF be drawn perpendicular to BC then AD : BC :: AF : 2XE, where XE = the radius of the inscribed circle, and AE : BC :: AF : 2YD, where YD is the radius of the escribed circle.

Next, if the area of the triangle be given, and the base, we can at once find AF, for make any rectangle having *double* the required area the lengths of the side of which are L and M then BC : L :: M : AF ; in other words AF is a fourth proportional to BC, L and M.

(1) To find the triangle ABC when its altitude, sum of sides and vertical angle are given.

Make the angle at A equal to the given vertical angle, and bisect it by the straight line AXY.

160.



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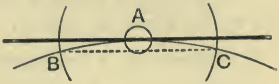

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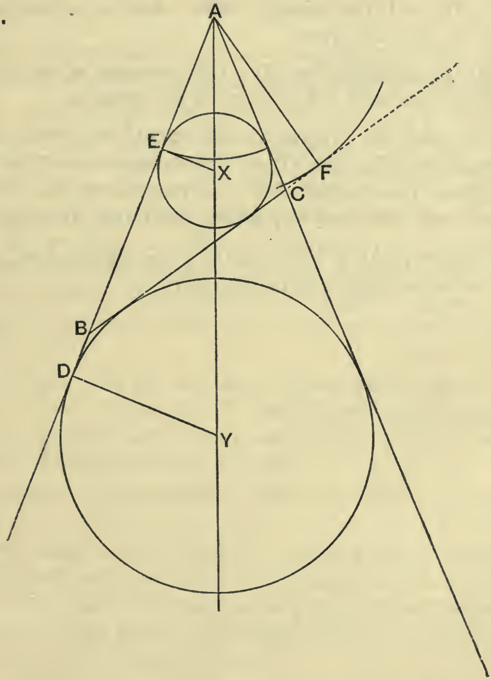

Measure a length AD = half the sum of the sides along one of the lines containing the angle.

At D draw a straight line at right angles to AD to meet AXY in Y, this gives the point Y.

With centre Y and distance YD draw the escribed circle.

With centre A and distance AF make another circular arc, this can be done, for the length of AF is given.

Draw a common tangent to these two circles between them. This tangent is the required base BC (66).

(2) To find the triangle ABC when its altitude, sum of sides and base are given.

Draw the straight line AD = half the sum of the sides, from D measure the base DE, this gives the point E.

Next take the proportion AE : BC :: AF : 2YD, we obtain the length of YD, draw YD at right angles to AD at D of the length thus found, then AC is a tangent to the circle, with centre Y and distance YD, drawn from the given point A.

As before make a circle centre A and distance AF, and draw BC a common tangent to the two circles.

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CHAPTER XVI.

SOLID GEOMETRY.

WHEN a person stands before a window and looks upon objects without, the different points upon these objects are seen by the eye along straight lines converging to a point in the eye. Now if these straight lines, called projectors, intersect the plane of the window in certain points we can, by representing these intersections by marks made on the window, obtain an exact representation of the objects as seen by the eye. This vertical plane of the window is called the picture plane. The line representing the intersection of this plane with the horizontal plane under the feet of the observer (the ground plane) is the ground line, sometimes called the intersecting line or axis, denoted by XY.

When objects are far off they appear smaller than when nearer the picture plane, sets of parallel lines appear each to converge to a point, and lines drawn on the picture are not proportional to their actual lengths.

This method of representing objects is called perspective projection.

But now suppose the observer to move further away from the picture plane, along a straight line perpendicular to it, and yet to have the power of clearly seeing the objects beyond it, the distances of which from the picture are small as compared with the distance of the eye from it.

All projectors will now be perpendicular to the picture plane, or nearly so; the lines on the picture when parallel to one another will be proportional to the actual lengths of the lines in the object, and the heights of all points above the ground line may be accurately measured.

The projectors are taken as perpendicular to the vertical plane.

This is called Orthographic Projection.

The picture so obtained is called an elevation of the object on the Vertical Plane.

By taking another ground line we obtain another elevation. *The heights of all points are still the same as in the first elevation* but their positions with respect to one another are changed.

The elevation of a solid determines the heights of all points on it above the ground plane, and its breadth measured parallel to **XY**.

It is clear however that an elevation of an object is not all that is required.

Suppose however the eye of the observer were placed up above the object, and a view of it obtained at a considerable distance. Now we can find the lengths and breadths but not the heights of the objects. This observation is called the plan of the objects. Its outline is made by drawing perpendiculars from every point of the object to the horizontal ground plane. These perpendiculars are also called projectors.

Take a rectangular solid **ABCGEDHF**, for example a brick hung up as represented in fig. 163.

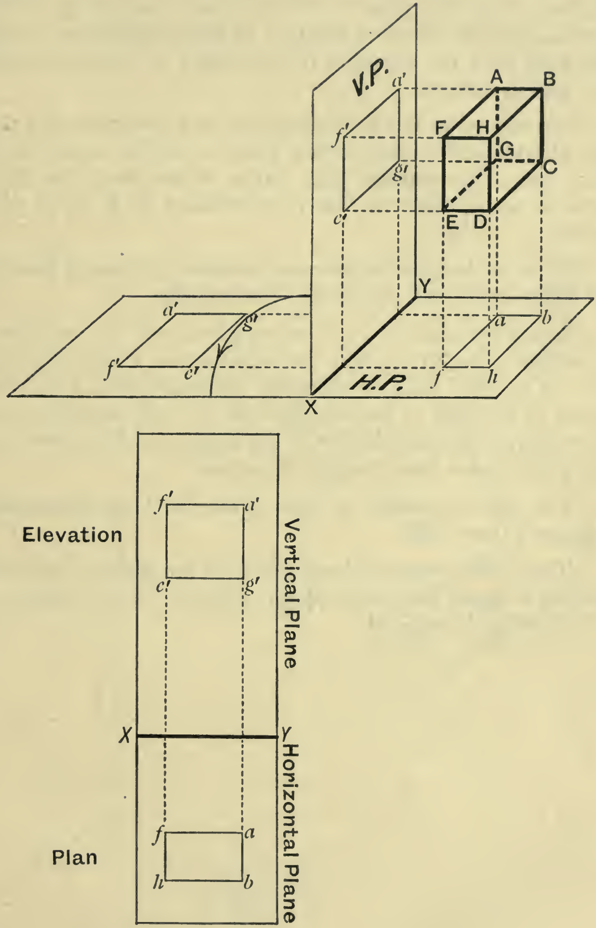
The plan of this solid is *abhf* and its elevation $a'f'e'g'$. **A** and **B** have the same elevation, namely a', **H** and **F** have the same elevation f', and so on; while **A** and **G** have the same plan, namely *a*, and so on for other pairs of points.

But **A** is distinguished from **B** or **G** for **A** has not the same plan as **B**, or the same elevation as **G**.

It is found more convenient not to represent the plan and elevation as in the first figure but to revolve the vertical plane (or V.P.) backwards in the direction of the arrow till it is in the same plane with the horizontal plane (or H.P.).

The drawing is then shewn as in the second figure (163).

163.


V.P.
a'
A
B
F
H
G
C
f'
g'
c'
E
D
Y
a'
g'
f'
c'
a
b
f
h
H.P.
X
Elevation
f'
a'
c'
g'
Vertical Plane
X
Y
Horizontal Plane
Plan
f
a
h
b

If the plan is given and the heights of the points A, B, C, G, E, D, F, H stated, we find the elevations by drawing perpendiculars from the plan to XY and then measuring off the projectors of the required length. In the same way we can find the plan from the elevation if the lengths of the projectors for the plan are stated.

The reason for this is that the plan and elevation of a point are always in the same vertical plane as will be seen from the first figure, for example (fig. 163) a, a' and A are all in one vertical plane which in this case contains A, B, G, C, a', g', a and b.

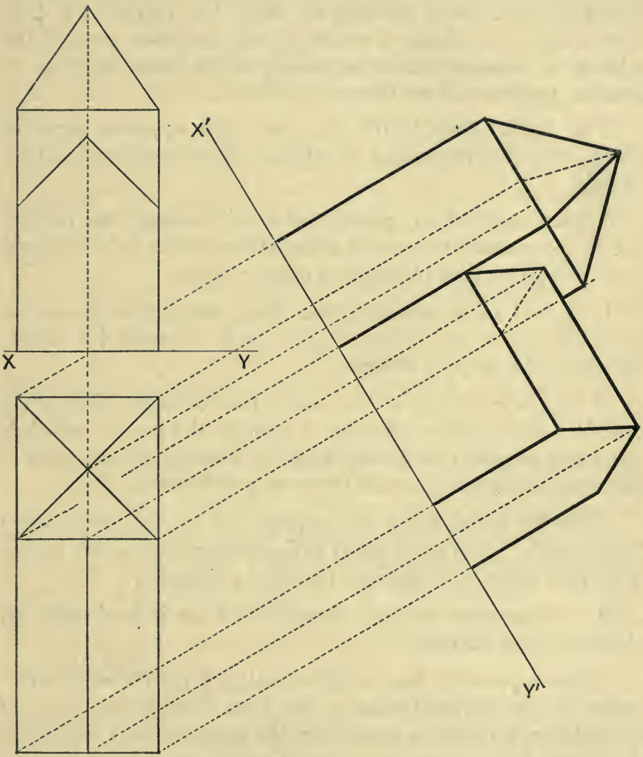
Since the heights in different elevations are always the same, as before explained, we get the following rule.

When the plan of an object is given, and its elevation on a straight line XY we find the elevation on a second straight line X′Y′ (fig. 164) by drawing perpendiculars from every point of the plan to the straight line X′Y′ and measuring along these perpendiculars above X′Y′ the heights of the elevations of the points taken from the first elevation.

Fig. 164 is similar to that given in "Art Examination Papers," June 1898.

The "block" plan and end elevation are given of a building having a square tower with pyramidal roof. A side elevation of the building is required.

164.



A cube is a solid figure each face of which is a square. It has six faces, and all its edges are equal.

Comparing this definition with the figure (163) of the solid **ABCGEDHF** we see that the plan and elevation of a cube in the same position are equal squares. If then the plan of a cube is given we can at once construct its elevation. If, however, the plan be turned round through an angle the elevation is now a rectangle, the height of which is still the same, namely the side of the square, but its breadth is found from the plan by drawing projectors from the plan to **XY**.

The figure **ABCGEDHF** (fig. 163) the opposite faces of which are equal rectangles is called a rectangular prism, block or slab.

Take a circle in any plane, and a point called "the vertex" not in the plane. Join every point of the circle to the vertex. The solid figure thus obtained is called a cone.

It is said to be a right cone when the vertex is on the perpendicular to the plane of the circle through its centre, otherwise the cone is oblique.

If the circle, or base of the cone is horizontal the plan of the cone is a circle and its elevation a triangle, the base of which is equal and parallel to a diameter of the circle, and the vertex of the triangle the projection of the vertex of the cone.

Take any plane figure and a point not in the plane called "the vertex." Join every point of the plane figure to the vertex. The solid figure thus obtained is called a pyramid.

As in the case of the cone if its base is horizontal its elevation is a triangle.

When a pyramid has a regular polygon for its base and its vertex on the perpendicular to the base through its centre of the polygon it takes its name from the polygon upon which it is described, for instance a square pyramid, on a square base; hexagonal pyramid, on a regular hexagon as base; and so on.

A prism is a solid contained by plane figures of which two are opposite, equal, similar and parallel to one another; and the others are parallelograms. When the parallelograms (not the parallel equal ends) are all rectangles the prism is said to be a right prism.

If one of the parallel equal ends of a right prism be horizontal the elevation of the prism is a rectangle the breadth of which is the same as that of the plan, but the height must be given.

A cylinder is a solid figure described by the revolution of a right-angled parallelogram about one of its sides which remains fixed.

The cylinder is like a prism the ends of which are parallel and equal circles, so that if one of its circular ends be taken as its plan the elevation is a rectangle the breadth of which is the diameter of the circle, and its height must be given.

A sphere is a solid figure described by the revolution of a semicircle about its diameter which remains fixed.

The plan and elevation of a sphere are equal circles, the straight line joining the centres of which is perpendicular to XY.

Any plane section of a sphere is a circle which varies in size according to the position of the plane of section but it will be found that the projection of this section on the horizontal or vertical plane is not a circle unless it be parallel to one of these planes, but is an ellipse. Similarly oblique sections of cones or cylinders and their projections are often ellipses. There is generally no difficulty in finding the greatest and least diameters of this ellipse which can then be described by methods given in Chapter XIV.

A pyramid on a triangular base is called a tetrahedron because it has four triangular faces, and when the four faces are equilateral triangles it is called a regular tetrahedron. In the same way a cube can be described as a hexahedron because of its six faces.

(165) Given the base of a regular tetrahedron BCD in the ground plane with one edge CD perpendicular to XY, to find its elevation.

When a pyramid has a regular polygon for its base, and its vertex on the line perpendicular to the base through the centre of the polygon, and *an edge* is known; find the centre O of the base, join O to C one of the angular points of the base; draw OV at right angles to OC, with centre C and distance the given edge, make an arc to cut OV in V, then OV is the height of the pyramid and therefore of the elevation of the pyramid.

In this case all the edges of the tetrahedron are equal so the arc is made with centre C and distance CD.

From the centre O of the base of the pyramid draw a perpendicular to XY, along this line produced measure a distance equal to OV, this gives A the elevation of the vertex of the tetrahedron. Join AB, AC. The points B and C are found in elevation by projecting from the plan on the line XY, for these points are on the base of the tetrahedron.

(166) If in the last proposition the tetrahedron be turned about its edge CD so that its base makes a given angle BCX with the ground plane find the new plan of the tetrahedron.

Draw the projectors from the new elevation perpendicular to XY and draw parallels to XY from the first plan. The projector of B meets the parallel from the first plan of B in the new position of the plan of B. The same method is used for other points. C and D have the same elevation in both figures, and in (165) O is the plan of A.

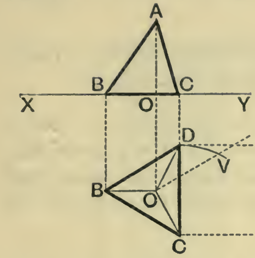
(167) Given the base ABCDE of a regular pentagonal prism in the ground plane, draw its elevation if its height be known.

From each angular point of the pentagon draw a perpendicular to XY and produce each projector to a distance equal to the given height of the prism.

(168) In the last proposition suppose the prism to be turned about its edge CD so that its base makes an angle NOX with the ground plane, find the new plan of the prism.

As in fig. 166 draw from each point in the new elevation projectors to XY and produce these to meet parallels to XY through their corresponding plans in fig. 167. Edges of the solid not seen, are given as dotted.

165.



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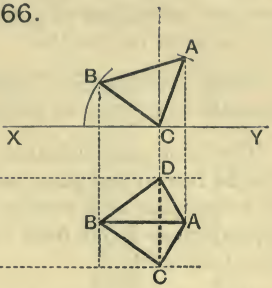

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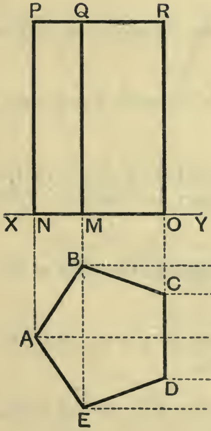

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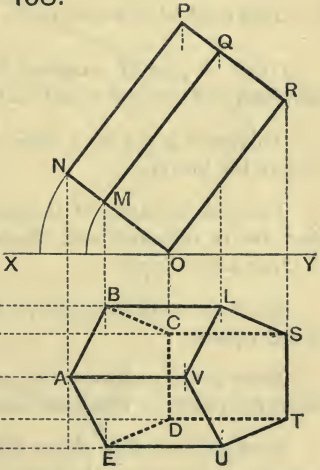


When sections of a solid are made by planes we find the plans and elevations of points bounding the planes of section and shade in the section.

When a section of a regular solid, cone, prism or pyramid is parallel to the plan or elevation we get a similar figure.

The plan and elevation of a solid being given, and a straight line in the plan, such as *lm* in fig. 169 representing the intersection of a vertical plane with the ground plane draw perpendiculars to XY from all points at which *lm* intersects the plan of the solid.

When these projectors are produced and points taken upon them which are their elevations we find a correct elevation of the section.

This is not the true shape of the section: to find the true shape draw perpendiculars to *lm* and from a line parallel to *lm* measure the elevations of all points. Joining these points we find the true shape of the section.

This method is shewn in fig. 173.

(169) A plan of a cubical block is given through the centre of which is a vertical cylindrical hole.

This solid is cut by a vertical plane which meets the ground line in the line *lm*.

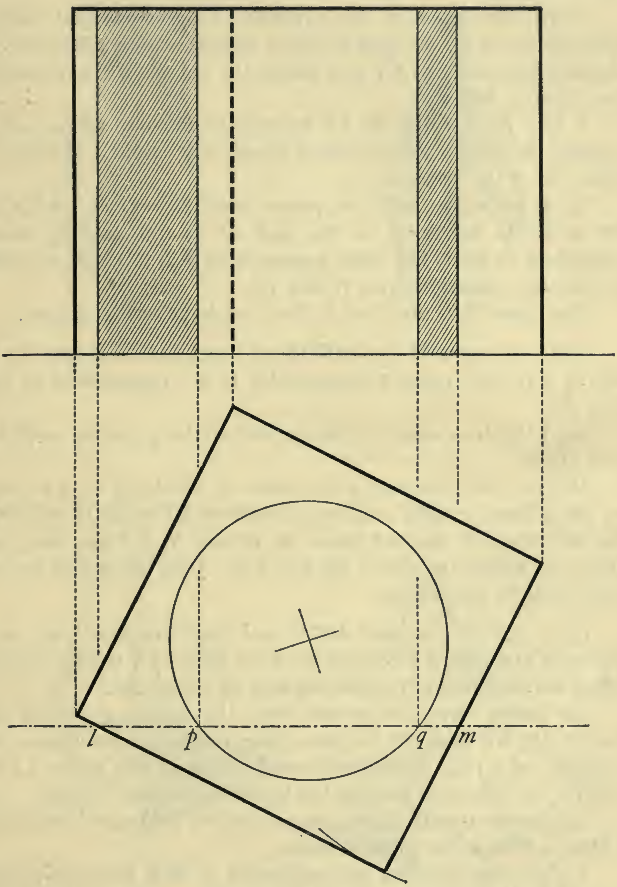
Find the elevation of the solid when the part in front of the line *lm* is removed and shade in the part in section. (Art Examination Papers.)

Since the solid is a cube its height is equal to the side of the given square.

Draw projectors from the angular points of the square, and from the points at which *lm* meets the solid.

Produce each line above XY equal to the side of the given square, and complete the figure as shewn.

169.



(170) Fig. 170 represents the plan and elevation of a pyramid the base of which is a regular pentagon, the edge CD being perpendicular to XY.

(171) The base of the pyramid in 170 is turned round through an angle as shewn, and a new elevation obtained by drawing projectors to XY and taking the height of the pyramid the same as before.

A line *lm* parallel to XY represents the intersection of a vertical plane with the horizontal plane, it is required to find the elevation of the section.

Draw projectors from the points at which *lm* meets AE, CD, OE and OD, where O is the plan of V, and produce these projectors to meet the lines representing AE, CD, OE and OD in elevation namely *ae*, *cd*, V*e* and V*d*.

The figure thus obtained is the true shape of the section.

(172) A pyramid on ABCDE as base, and of height V*o* is cut by a vertical plane perpendicular to XY represented by the line *lm*.

Find the true shape of the section of the pyramid made by this plane.

Measure the heights in elevation at which *lm* cuts V*b* and V*c* along lines at right angles to *lm* where it meets OB and OC, for OB and OC are the plans of V*b* and V*c*. Take also the points at which *lm* meets AB and CD. Join these four points and shade in the section.

(173) Given the plan ABCD and the elevation V*adbc* of a pyramid and *lm* the intersection of its plan by a vertical plane. Find the elevation of the section and its true shape.

As before draw projectors from the points at which *lm* meets AB, OB and CD to meet their respective elevations *ab*, V*b* and *cd*. Join these points and shade in the figure LMN. This is the elevation but not the true shape of the section.

To get the true shape of the section we find an elevation of LMN on M′N′ a line parallel to *lm*.

To get this we draw perpendiculars to M′N′ from the points at which *lm* meets AB, OB, CD and measure the height of L′ from M′N′ the same as that of L from MN.

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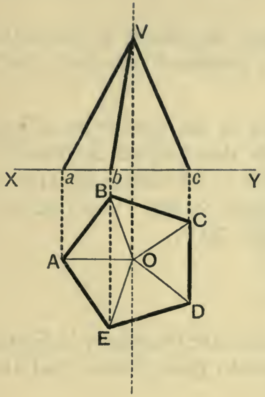

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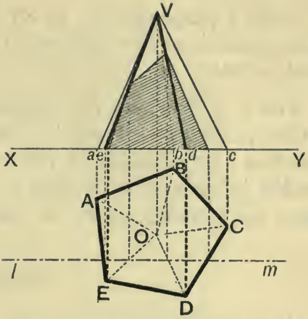

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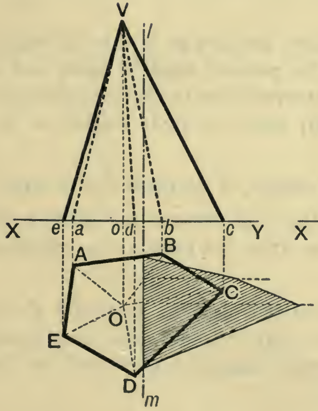

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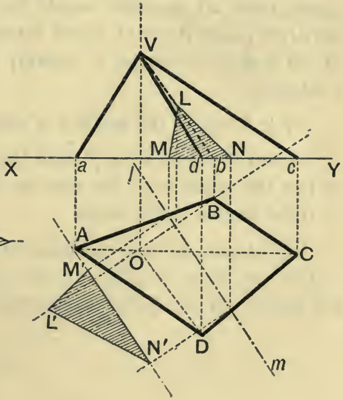


(174) **ABC** represents the plan of a portion of a sphere. Draw its elevation on the straight line **XY.** (Art Examination Papers, June 1898.)

Draw a perpendicular to **XY** from the centre of **ABC** and with a point as centre upon this line produced describe a circle equal to **ABC.**

Now it is clear that the true shape of the section made by a vertical plane through **AB** is a circle that has **AB** as diameter. Therefore the elevation of this circle is an ellipse (p. 143) its breadth is *ab* and its height is equal to **AB.** Describe this ellipse by methods shewn in Chapter XIV.

Indices.

On a map the heights of mountains are frequently indicated by numbers. 2000 placed by a certain point shews that this point is 2000 feet above sea level.

Lines are drawn also to indicate that all points upon them are at the same height above sea level, these lines are called contours.

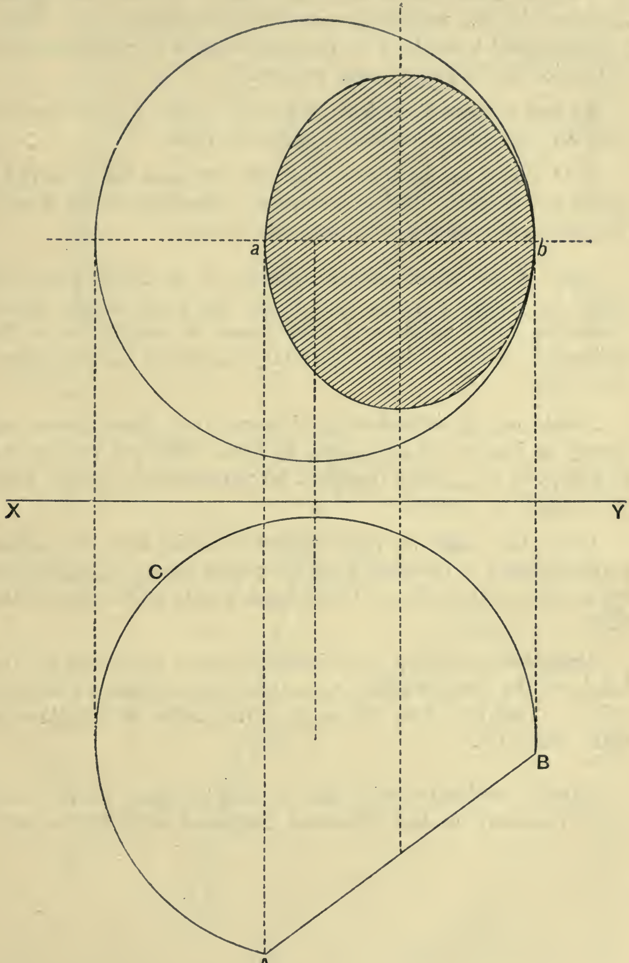
Now suppose instead of uneven country we had to do with a plane, then all contours would be parallel straight lines, and if we drew these lines at equal intervals (say to represent heights of 10 feet) we obtain a regular scale at right angles to the contours.

This is called the method of indices, or horizontal projection.

When we deal with planes the contours are not always put in but the scale only, for we can draw the contours as required at right angles to the scale.

We obtain the true position of the plane by turning it about a line at right angles to the scale through the zero point till points on the plane occupy the heights represented by the

174.



(175) Let XY be the scale of a plane representing that X is at a certain level called zero, and that Y is 100 feet higher. Suppose also YZ represents 100 feet to the same scale. Make a right-angled triangle XYZ then the angle ZXY represents the inclination of the plane to the horizon.

To find a point P 60 feet high we take the point 60 on the scale XY and draw a vertical to meet XZ in P.

If M and N are both at a height 80 feet then MN is at right angles to the scale. MN is a contour. Similarly R and S both at a height of 70 feet are on the same contour.

(176) Draw an equilateral triangle of 2½ inches side, and figure its points a_{15}, b_{40}, c_{35}. Draw the scale of the plane containing these points and write down its inclination to the horizontal. Unit 0·1 inch. (Military Entrance Examinations, Nov. 1898.)

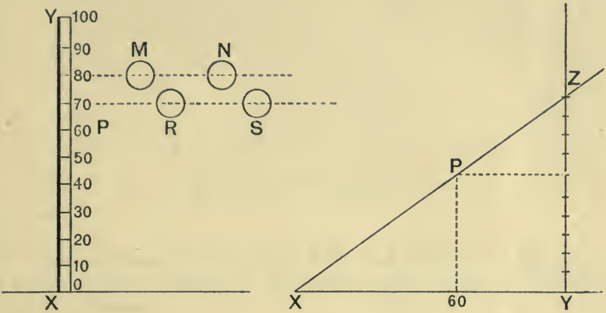
Divide a_{15}, b_{40} into five equal parts, then these points are clearly at heights of 20, 25, 30, 35 feet. The line joining the 35 feet point to c_{35} must therefore be perpendicular to the scale at the point 35 upon it.

Draw the scale at right angles to this line, and draw perpendiculars to the scale from the points on a_{15}, b_{40} at heights 15, 20, 25, 30, 35 and 40. These mark points of division on the scale.

Make AB measuring 10 divisions from the scale, and BC the height of the point B above A, in this case 10 times 0·1 inch, or 1 inch. Join AC, then the angle of elevation of the plane is CAB about 47°.

[*Army candidates will find a Chapter upon Indices and Solid Geometry in the Woolwich Text-book of Fortification.*]

175.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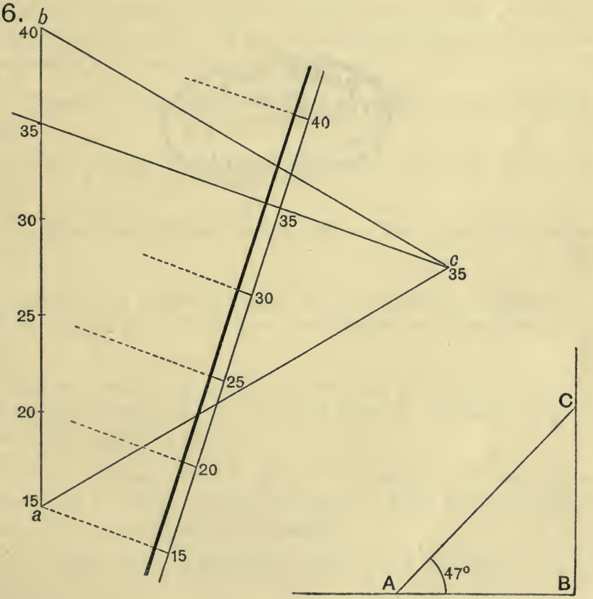
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EXAMPLES.

1. FIND and mark three points, **A**, **B**, and **C**, each of which is to be 2 inches from the other two. Join these points, and state what kind of triangle is formed. (P)

2. From any point **Z** draw two straight lines at any angle with each other between 60° and 90°. At any point **X** construct an angle equal to the angle at **Z** and bisect it. (P)

3. Construct a rhombus having sides of 2½ inches and two angles of 60°. (P)

4. Draw a plain scale of 2 inches to one foot, to shew feet and inches. (P)

5. A square has sides of 9 inches; draw it to the scale of 2 inches to a foot. (P)

6. Construct a regular hexagon of 2 inch sides. Draw all the diagonals, and in each of three of the triangles so formed inscribe a circle. (P)

7. Draw a straight line **AB** 2 inches long, and on one side of it find a point **C** 2 inches from **A** and 1 inch from **B**. Through the point **C** draw a line parallel to **AB**. (P)

8. Draw a straight line **XY** 1½ inches in length, and upon this as base construct a triangle having the angle at **X** 30° and the angle at **Y** 120°. What kind of triangle is formed? (P)

9. Construct a rectangle having sides of 2 inches and 1¼ inches respectively.
Find its centre and mark it **C**. (P)

10. Construct a square of 1 inch sides and draw its diagonals; at each of its angles draw a line at right angles to the diagonal and produce these lines to form a second square. (P)

11. Draw a straight line 6 inches in length. This line represents 3 feet; divide it to shew feet and inches, and complete and figure the scale. (P)

12. Construct an equilateral triangle of 3 inch sides, bisect each of its angles and produce the bisecting lines to meet in the centre of the triangle.

In each of the three triangles thus formed inscribe a circle. (P)

13. Describe a triangle the sides of which are 2 inches, 3 inches, and 4 inches. Bisect the greatest angle of the triangle, and measure and write down the lengths of the segments into which the bisecting line divides the opposite side. (C)

14. One angle of a rhombus is 135°, and a side 1·5 inches. Find the mean proportional between its diagonals. (C)

15. Within a circle of radius 2 inches, describe a regular heptagon. (C)

16. Three hundred yards are shewn by 4·5 inches. Construct a diagonal scale to shew single yards. (C)

17. The major axis of an ellipse is 5 inches, and the minor axis 4 inches; draw a quadrant or fourth part of this ellipse. (C)

18. Draw a square on a straight line 3 inches long. Join its diagonals. In each of the four triangles thus formed inscribe a circle, and in each of these circles place a square having its sides parallel to those of the original square. (C)

19. Describe a triangle two sides of which are 2 inches and 3 inches and the included angle 30°. Bisect the shortest side of this triangle by a straight line at right angles to it, and produce the bisecting line to meet the other two sides or sides produced of the triangle.

Measure and write down the lengths of the segments into which this bisecting line is divided by the sides. (C)

20. Construct an isosceles right-angled triangle, hypotenuse 2 inches. Draw a perpendicular from the right angle to the opposite side. Find a third proportional to the hypotenuse and this perpendicular. (C)

21. About a circle of 1·5 inches diameter describe a regular pentagon. (C)

22. Four hundred paces being represented by 5 inches, construct a diagonal scale to shew single paces. (C)

23. The major axis of an ellipse is 4 inches, and 3 inches is the distance between its foci. Draw half of this ellipse. (C)

24. Draw an equilateral triangle on a straight line 4 inches long. Join the points bisecting the sides, so as to divide the original triangle into four equal parts. In each of these triangles inscribe a circle, and draw in each circle a triangle, having its sides parallel to those of the original triangle. (C)

25. Construct a triangle the perimeter of which is 5 inches, its angles being in the proportion 2 : 3 : 4. (C)

26. Find a mean proportional between the side and half the diagonal of a square having sides of 1·5 inches. (C)

27. Construct a regular octagon on a base of ·75 inch. (C)

28. Eighty yards being represented by six inches, construct a scale, shewing single yards, to measure distances up to sixty yards. (C)

29. Draw an ellipse the major axis of which is 4·5 inches in length and the minor axis 3·5 inches. (C)

30. Describe a circle the diameter of which is 4·5 inches. Divide this circle into seven equal sectors, and in each sector inscribe a circle, to touch the original circle, and two of the other inscribed circles. (C)

31. Construct a triangle the perimeter of which is 5 inches, its sides being in the proportion 6 : 7 : 8. (C)

32. Find a mean proportional between the altitude and half the base of an equilateral triangle, the side of which is 3 inches. (C)

33. Construct a regular pentagon on a base of 2 inches. (C)

34. Seven feet are shewn by six inches. Construct a scale, shewing feet and inches, to measure distances up to five feet. (C)

35. Draw an ellipse the major axis of which is 4 inches in length and the minor axis 3 inches. (C)

36. Draw an equilateral triangle on a straight line 3·5 inches long. Divide each of its sides into three equal parts. By joining the points thus found to each other by straight lines parallel to the sides we obtain nine equal equilateral triangles. Inscribe a circle in each of these triangles. (C)

37. Construct a rectangle 2½″ × 1¾″. Within it inscribe two other rectangles, each similar to the first, concentric with it and having their longer sides 1½″ and 1″ long respectively. (S)

38. Construct a regular pentagon of 1½″ side. Describe five circles of ¾″ radius, having their centres at the five angles of the pentagon. (S)

39. Describe two circles, each of ½″ radius, touching each other at a point A. Find a point B on one of the circles, ⅞″ from A. Describe a third circle, touching both the others and passing through B. (S)

40. Construct a triangle, sides 1½″, 2″, and 2½″, and within it inscribe an equilateral triangle having its three angles in the three sides of the first triangle. (S)

41. Divide a line AB into three consecutive parts to each other in the proportion of 1 : ¾ : ½. (S)

42. Within a circle of 2¾″ diameter, inscribe four equal circles each touching the given circle and two of the others. (S)

43. Construct a scale one-tenth of full size, to measure feet and inches up to 5 feet. The scale must be properly finished and figured. (S)

44. Draw the figure shewn, using the figured dimensions. [177] (S)

45. Draw the "cyma recta" moulding shown in the diagram, using the figured dimensions. The curve is composed of two equal tangential arcs each of ¾″ radius. [178] (S)

46. A circle of 1″ radius rolls along a straight line. Draw the curve (cycloid) traced by a point on the circumference of the circle during half a revolution. (S)

47. Draw the figure from the indicated dimensions. [179] (S)

48. Construct and figure correctly a scale of ·8″ to 10′ 0″, capable of giving dimensions up to 90′ 0″. (S)

49. Construct a triangle with sides 3″, 2¼″, and 1¼″. Construct a similar triangle with a perimeter of 8½″. (S)

50. A line 4¾″ long represents a distance of 4′. Construct a scale by which feet and inches may be measured up to 4 feet. The scale must be neatly finished and correctly figured. (S)

51. Within a circle of 1¼″ radius inscribe a regular pentagon. About the same circle describe another regular pentagon, having its sides parallel to those of the inscribed pentagon. (S)

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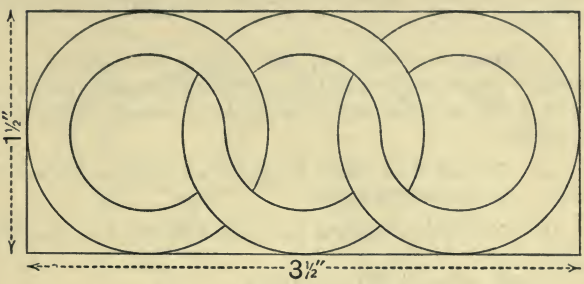

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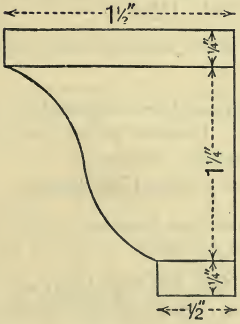

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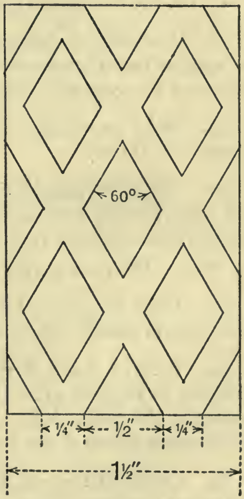

52. Construct a rectangle having one of its sides $1\frac{5}{8}''$, and its diagonals $2''$ long. Make a similar rectangle having its shorter sides $1\frac{1}{2}''$ long. (S)

53. Draw two lines **AB**, **AC**, making an angle of 25° at **A**. Describe a circle of $\frac{3}{4}''$ radius touching **AB** and having its centre on **AC**. From **A** draw a second tangent to the circle, marking clearly the point of contact. (S)

54. The foci of an ellipse are $2\frac{1}{2}''$ apart and its major axis is $3\frac{1}{2}''$ long. Describe half the curve. (S)

55. Construct an irregular pentagon, **ABCDE**, from the following data:

Sides—**AB** $= 2''$, **BC** $= \frac{3}{4}''$, **AE** $= \frac{3}{4}''$.

Angles—**ABC** and **BAE** each $= 135°$. **BCD** and **AED** each $= 95°$.

Reduce the figure to a triangle of equal area having **AB** (produced if necessary) for base and **D** for vertex. (S)

56. Draw the curve every point of which is at equal distances from a line **PQ** and a point **F**. The curve is a parabola. (S)

57. Copy the pattern given. [180] (A)

58. The diagram represents an incomplete scale of feet. Complete the scale so that distances of $2'$ may be measured by it up to $50'$. The scale must be properly finished and figured. [181] (S)

59. Make an enlarged copy of the given diagram, using the figured dimensions. [182] (S)

60. Draw two parallel lines $\frac{3}{4}''$ apart, and cut at an angle of 75° by two other parallel lines also $\frac{3}{4}''$ apart. Describe all the circles of $\frac{5}{8}''$ radius which touch any two of these lines and cut none.

N.B.—The lines are supposed to be produced indefinitely. (S)

61. Draw two parallel lines $1\frac{1}{4}''$ apart and describe a circle cutting these lines in chords respectively $1''$ and $1\frac{5}{8}''$ long. (S)

62. From a point **O** equal lines **OA**, **OB** ($2\frac{1}{2}''$ long) are drawn including an angle of 50°. Describe the circle to which these lines are tangent at the points **A, B**. On **OA** produced determine a point such that tangents drawn to the same circle shall include an angle of 70°. (S)

63. Construct a square equal to the difference of the areas of two equilateral triangles whose sides are $3''$ and $1\frac{3}{4}''$ respectively. (S)

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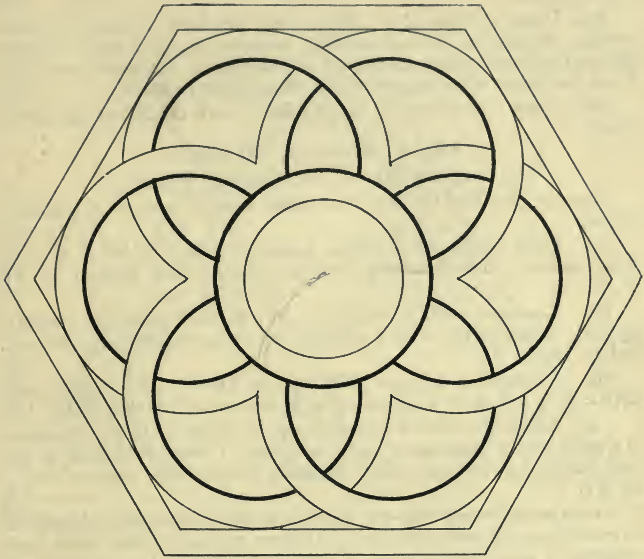

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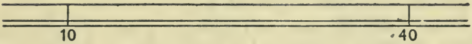

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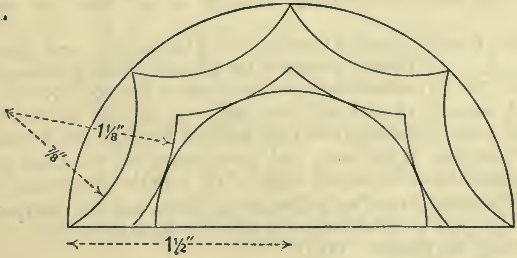


64. Inscribe a pentagon in a circle of 1·5″ radius. Draw the diagonals, and the circle circumscribing the pentagon formed by their intersections. (S)

65. Draw a circle of 1″ radius; and (*a*) draw a triangle circumscribing the circle, its angles being, respectively, 75°, 70°, and 35°; (*b*) join the points where these lines are tangent to the circle, and write down the values of the angles of the triangle thus formed. (S)

66. Draw a quadrilateral figure **ABCD**, with the following dimensions:

$$\mathsf{AB} = 2'', \ \mathsf{BC} = 1{\cdot}5'', \ \mathsf{AD} = 1{\cdot}5''.$$

The diagonal **BD** = 2″, the diagonal **AC** = 2·5″.

Find the length of the side of a square equal in area to the quadrilateral. (S)

67. Draw a circle of 1″ radius, tangent to the line **AB** at **Q**; and draw another circle touching the first circle, and tangent to **AB** at **P**. (S)

68. Construct a scale of 3·5″ to 30′, to shew feet, and correctly figured. Draw to the scale a triangle, its sides, respectively, 20′, 15′, and 12′ long. (S)

69. Construct a triangle **ABC** having **AB**=2″, **AC**=1·75″, angle **CAB**=40°; and draw a square equal in area to the triangle **ABC**. (S)

70. A plan drawn to a scale the R.F. of which is $\frac{1}{11}$ is to be copied to a scale half as large again as the original. Construct a scale of feet and inches for the new plan (shewing 4 feet) and write above it its R.F. (A)

71. On a French map the distance between two points known to be exactly one kilometre (3280·9 feet) apart measures $\frac{7}{10}$ths of an inch, construct a comparative scale of miles and furlongs for the map and give its R.F. (A)

72. Upon a straight line 2 inches long, construct an isosceles triangle having a vertical angle of 35°. (A)

73. Draw a straight line, and taking a point 1½ inches distant from it as centre, describe a circle of 1 inch radius. Draw a straight line touching this circle, and making an angle of 40° with the straight line. (A)

74. Draw an equilateral triangle, of 2½ inches side, and figure its points a_{15}, b_{40}, c_{35}. Draw the scale of the plane containing these points, and write down its inclination to the horizontal. Unit 0·1 inch. (A)

75. A cube of 1½ inch side rests with one edge in the horizontal plane, and a face containing that edge inclined at 32° to the horizontal.

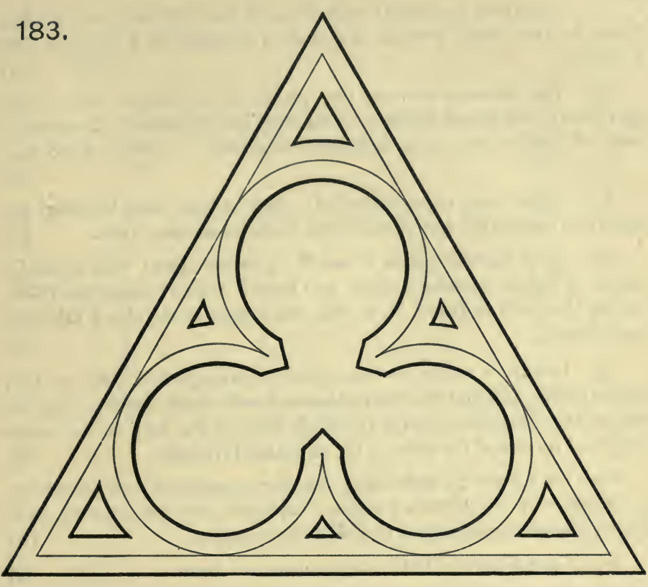
Draw the plan of the cube, and an elevation on a vertical plane, whose horizontal trace makes an angle of 15° with the given edge. (A)

76. The minor axis of an ellipse is 2¼″ long, and the distance between the foci is 2″. Find the major axis and draw the curve. (S)

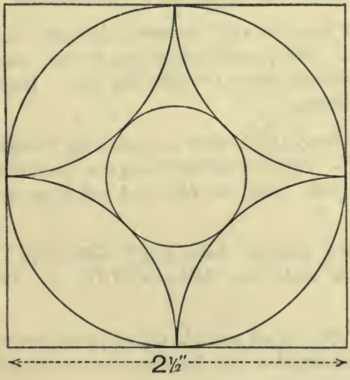
77. Copy the pattern. [183] (A)

78. Draw the figure shewn, making the sides of the square 2½″ long. [184] (S)

183.



184.



79. Construct a diagonal scale of yards, feet, and inches, R.F. $\frac{1}{120}$. Shew by two small dots on the scale a distance of 8 yards 2 feet 7 inches. (A)

80. The distance between two places on a Russian map scales 3¾ versts. The actual distance on the map is 2·85 inches. Construct a scale of English miles and furlongs for the map. 1 verst = 3,500 feet. (A)

81. Draw three equal circles of 1-inch radius, each touching the other two externally, and circumscribe them by another circle. (A)

82. Take any two points **C** and **P**, 2¾ inches apart; with centre **C**, radius 1¼ inches, describe a circle, and from **P** draw a straight line **PQR**, cutting the circle in **Q** and **R**, so that the length of the chord **QR** may be 1½ inches. (A)

83. Inscribe a square and an equilateral triangle in a circle of 1·73 inches radius, and find by construction and write down the ratio that the area of the square inscribed in the circle bears to the area of the square described on one of the sides of the equilateral triangle. (A)

84. On a line 1·75 inches long describe by means of your protractor, or in any way you please, a regular heptagon, and construct an equilateral triangle equal in area to ⅓rd of the heptagon. (A)

85. Copy diagram [185] using dimensions shewn. (S)

86. Copy the diagram according to the given dimensions. Shew clearly how the centre of the small circle is determined. [186] (S)

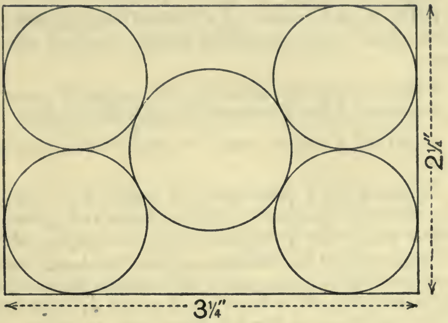
87. Draw the plan of a right pyramid 2½″ high, base an equilateral triangle of 2″ side. The pyramid stands on its base, and the upper part is cut off by a horizontal plane 1″ above the base. Indicate the section clearly by light shading. (S)

88. Draw a triangle **ABC** with the following dimensions: **AB** = 4″, **BC** = 2¾″, **AC** = 2″. Inscribe in the triangle a parallelogram with four equal sides, one side lying on **AC**, and adjoining sides inclined to **AC** at 45°. (S)

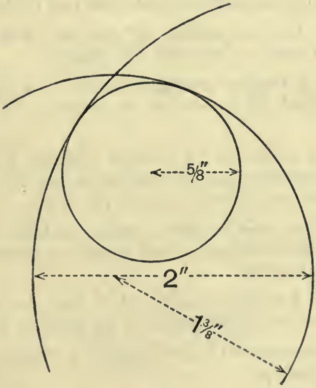
89. Given two lengths, $AB = 2·25''$, $CD = 2·70''$. Find a line whose length x is such that $AB^2 = (x - CD) \times x$. Write down the value of x. (S)

90. Draw a right-angled triangle with hypotenuse 2″ and one side 1¾″. Prove by construction that the square on the hypotenuse is equal to the sum of the squares on the sides. (S)

185.



186.



91. Construct a square **ABCD** of 2½″ side. Bisect this square by a line drawn from a point on the side **AB** distant 1″ from **A**. Reduce this square to a parallelogram of which the bisecting line is a diagonal. (S)

92. Draw a line **AB**, 2″ long. Describe a circle of ¾″ radius touching **AB** at **A**, and another of 1″ radius touching **AB** at **B**. Draw a second line which shall touch both circles, shewing clearly the points of contact. (S)

93. Within a rhombus, sides 2½″, one angle 60°, inscribe an ellipse touching the sides of the rhombus at their middle points. (S)

94. Construct a square equal in area to an equilateral triangle of 2″ side. (S)

95. Construct (*a*) a plain scale of yards, R.F. $\frac{1}{32000}$, to read to 100 yards, and (*b*) a comparative scale of miles and furlongs. [*The fair calculations must be clearly written on the same page as the scales.*] (A)

In making, without instruments, a plan of a small piece of ground, a man assumes that he paces yards, and extemporizes a scale of 10 yards to an inch by marking off on the edge of a card what he considers inches and dividing the inch to the left hand into 10 parts. On returning home he finds he has overestimated the length of an inch by 0·15 inches, and that instead of pacing yards the length of his pace is 34 inches. Construct an accurate scale of yards for the plan, and write above it its R.F. correct to the second decimal place. [*The fair calculations must be clearly written on the same page as the scale.*]

96. Inscribe a regular octagon in a square of 2½ inches side. (A)

97. Describe a circle of 3 inches diameter, and place in it six equal circles, each touching the original circle and two of the others. (A)

98. A triangle has two angles, 47° and 66°, and the radius of the inscribed circle is 1 inch. Construct the triangle. (A)

99. Construct a regular pentagon equal to a figure **ABCDEFG**. (The angle subtended by a side at the centre of a regular pentagon is 72°.) (A)

100. Within a circle of 1½″ radius inscribe a regular nonagon. Within the nonagon inscribe a rectangle having all its angles in the sides of the nonagon, and one of its sides 1½″ long. (S)

101. Construct a scale to shew yards and feet, on which 3½″ represent 8 yards. Make the scale long enough to measure 10 yards and finish and figure it properly. (S)

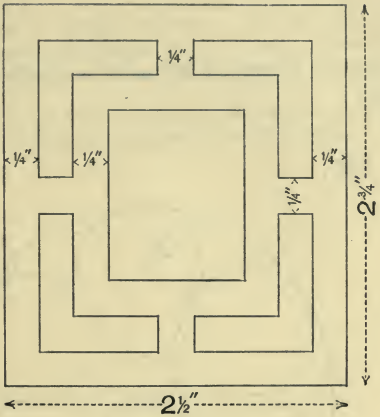
102. Construct a regular octagon of 1″ side, and a second octagon having its angles at the middle points of the sides of the first. (S)

103. About a square of 1″ side describe a triangle having one of its angles 60° and another 70°. (S)

104. Describe the figure to the dimensions given. [187] (S)

105. Describe a regular hexagon of 1¼″ side, and complete the figure. [188] (S)

187.



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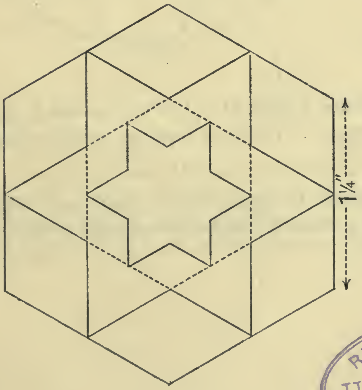

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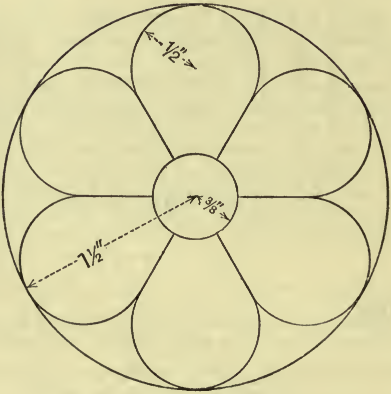
106. Describe a circle of 1½″ radius and within it complete the figure. [189] (S)

189.



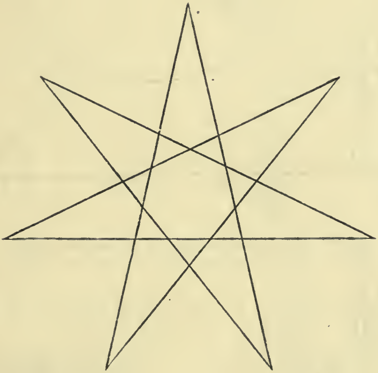
107. Construct a scale of 7½′ to 1″, by which single feet may be measured up to 30′. The scale must be neatly finished and correctly figured. (S)

108. Construct a square of 1½″ side, and within it inscribe a rectangle having one of its angles on each side of the square and one of its sides 1″ long. (S)

109. Construct a triangle, sides 1½″, 1⅞″, and 2⅛″ long, and a similar triangle having its longest side 2½″ long. Measure and write down the number of degrees in each of the angles. (S)

110. Draw a figure similar to the one shewn, the points of the star being at the angles of a regular heptagon inscribed within a circle of 1¼″ radius. [190] (S)

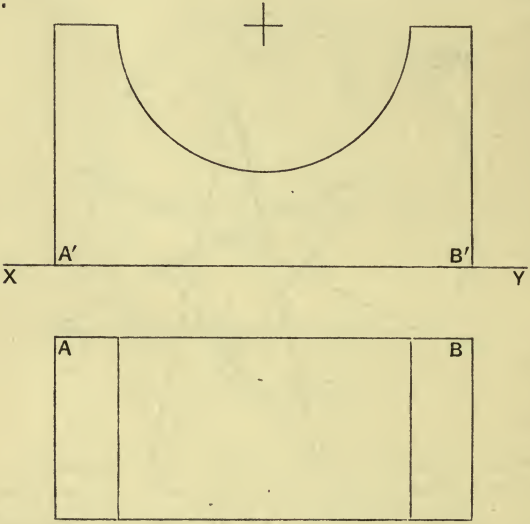
190.



111. Draw two straight lines **AB**, **AC** each about 4″ inclined at an angle of 30°. Describe a circle ¾″ radius to touch **AB** and **AC** in **D** and **E**. Describe a second circle to touch the first circle and **DB** and **DC** in **L** and **M**. Describe a third circle to touch the second and **LB** and **MC**.

112. Plan and elevation are given of a rectangular block with a semi-cylindrical hollow in it. Draw a new elevation upon a vertical plane which makes an angle of 30° with the horizontal edge **AB**. [191] (S)

191.



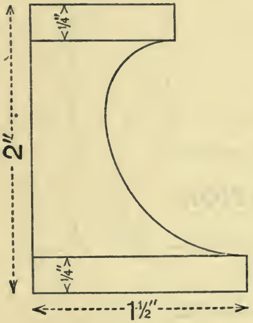
113. Within an equilateral triangle of 3″ side inscribe three equal circles, each touching the two others and two sides of the triangle. (S)

114. Draw the "Scotia" moulding shewn. The curve is made up of two quarter-circles of 1″ and ½″ radius respectively. [192] (S)

115. Describe the involute of a circle of 1″ diameter, *i.e.*, the spiral curve traced by the end of a string which is kept stretched as it is unwound from the circumference of the circle. (S)

116. Draw the moulding shewn, adhering strictly to the figured dimensions. The arc of ½″ radius is a quadrant. [193] (S)

192.



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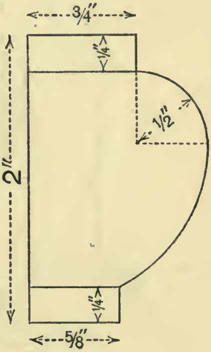
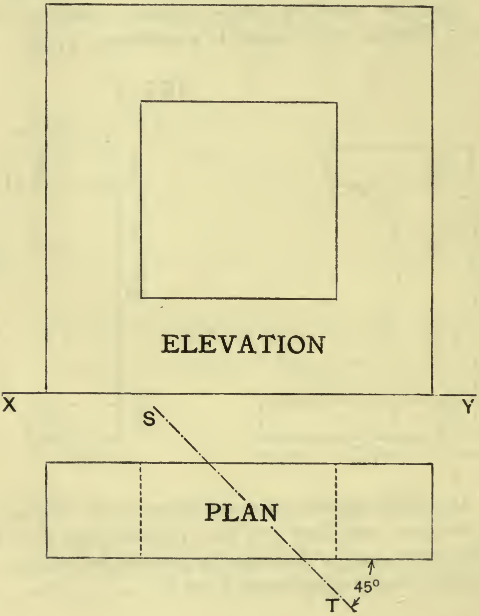


117. An arch in the form of a semi-ellipse is 6′ wide and 2′ high. Describe the curve, and draw two lines perpendicular to it from two points on the curve, each 2′ from the top point of the arch.

Scale (which need not be drawn) ½″ to 1′. (S)

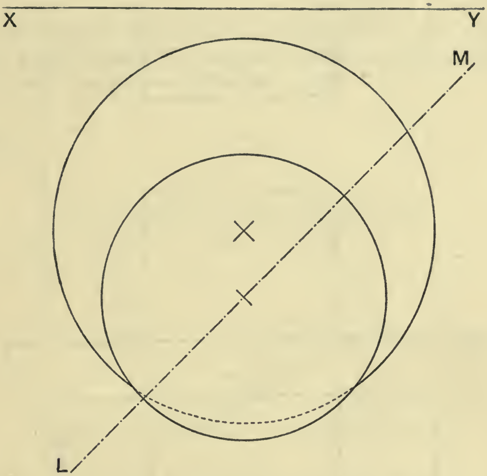
118. Plan and elevation are given of a square frame. Make a new elevation on a plane parallel to the line **ST**, and shew on it the section made by the vertical plane of which **ST** is the plan. The part in section must be clearly indicated by lightly shading it. [194] (S)

194.



119. The plan is given of a cylindrical slab $\frac{5}{8}''$ thick, and of a sphere resting on the slab. Draw an elevation of the solids on the line *xy*, and also the elevation of the section made by the vertical plane represented in plan by the line LM. The part in section must be clearly indicated by lightly shading it. [195] (S)

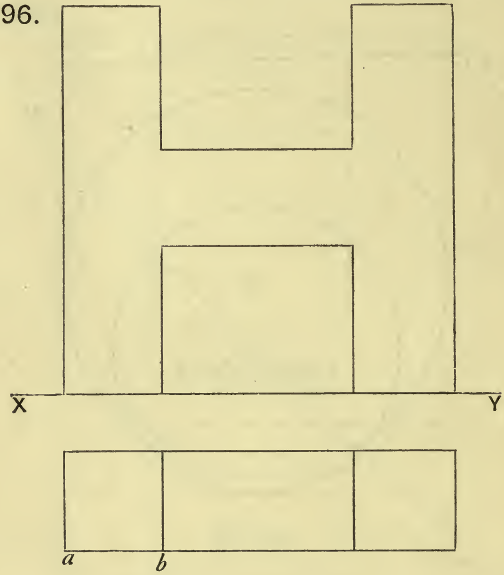
195.



120. A right prism, 2″ long, and having for base a scalene triangle, sides 1″, $1\frac{1}{4}''$ and $1\frac{1}{2}''$, lies with its smallest rectangular face on the ground. Draw the plan of the prism. (S)

121. Plan and elevation are given of a solid letter **H**. Draw an elevation when the horizontal edge *ab* makes an angle of 60° with the vertical plane of projection. [196] (S)

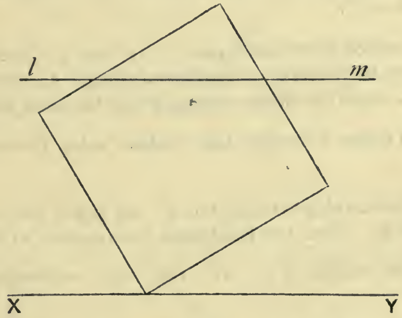
196.



122. A sphere of 1″ radius has a portion cut off by a horizontal plane $\frac{5}{8}$″ above its centre, and another portion by a vertical plane passing $\frac{1}{2}$″ from the centre. Draw a plan of what remains of the sphere. The section must be clearly indicated by lightly shading it. (S)

123. The diagram shews the end elevation of a right prism $1\frac{3}{4}$″ long with square base, and a horizontal plane *lm* cutting the prism. Draw a plan of the portion below *lm*. [197] (S)

197.



124. The plan is shewn of three bricks, each $9'' \times 4\frac{1}{2}'' \times 3''$, one resting upon the other two. Draw an elevation upon the given line *xy*. [198] (S)

Scale (which need not be drawn) 2″ to 1′, or $\frac{1}{6}$ of full size.

125. Draw the figure to the dimensions given. [199] (S)

126. Draw a triangle of $1\frac{1}{2}''$, $2''$, and $2\frac{1}{2}''$ sides and make a parallelogram equal to it in area and perimeter. (S)

127. Describe a circle 2″ in diameter, and within it place 5 equal circles each to touch it and two of the inscribed circles. (S)

128. Draw a triangle of $2''$, $2\frac{1}{2}''$ and $3''$ sides and find in the longest side a point equally distant from the other two sides. (S)

129. Take two points A and B, and through a third point C draw a straight line equally distant from A and B. (S)

130. In a square the side of which is 3″ inscribe a rectangle one side of which is 2″. (S)

131. Describe three circles radii $1'$, $1\frac{1}{2}''$ and $2''$ the centres of which are at the angular points of an equilateral triangle $3\frac{1}{4}''$ side. Draw two circles one to touch the circles externally and the other internally. (S)

132. To trisect a straight line, without using the method of proportion. (S)

133. Draw an ellipse major axis 3″ and minor axis 2″ and take a point without it. From this point draw two tangents to the ellipse. (S)

134. Make angles of 70°, 95°, 110°, 135°, without using the protractor. (A)

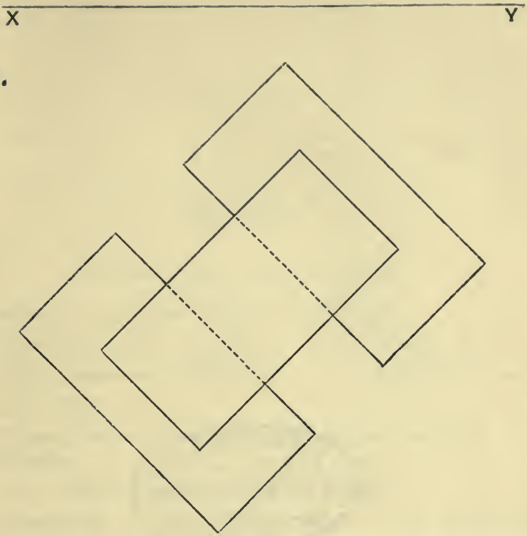
135. On a straight line 2″ long describe isosceles triangles having vertical angles of 100°, 120°, 150°. (A)

136. Draw a sector of 100° radius 2″, and within it inscribe a circle.

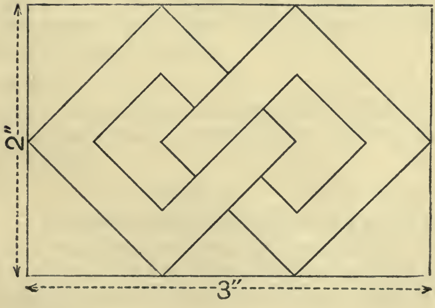
137. Draw straight lines the lengths of which are $\sqrt{3}''$, $\sqrt{\frac{1}{3}}''$ and $\frac{2}{\sqrt{3}}''$. (A)

138. Describe a triangle the vertical angle of which is 40°, the sum of its sides equal to 5″ and its height 2″.

198.



199.



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NOTES.

(*Bracketed numbers refer to figures.*)

1. **ABC** is an equilateral triangle on 2″ side (5).

2. (29) and (1). 3. (11). 4. Chapter X.

5. Draw a square on a side $1\frac{1}{2}''$ (9). 6. Chapters V and IX.

7. (7) and (15).

8. (22) and note on obtuse angles, the triangle is an obtuse-angled isosceles triangle.

9. (14). Join the diagonals to find the centre.

10. (9) and (19). 11. Chapter X. 12. (5) and (47).

13. (7) and (1). The greatest angle is opposite to the greatest side.

14. (11) and (42). 15. (101), (103) or (104).

16. Chapter X. 17. (127) to (130).

18. (9), (47) and in each circle draw diameters parallel to the diagonals of the original square, these diameters meet the circles in the angular points of the squares required.

19. (7) and (2).

20. Describe a semicircle on a straight line 2″ in length, bisect the semicircle by a straight line at right angles to the first through the centre. Then use (40).

21. (107). 22. Chapter X. 23. (127) to (130).

24. (5). To inscribe an equilateral triangle in a circle, step off the radius round the circumference and take alternate points. Chapter IX. If the original triangle is on a horizontal line the vertex of each of the inscribed equilateral triangles will be at the highest point of the circle in which it is inscribed.

25. Construct a semicircle on any straight line and divide it by trial into nine equal parts. Join to the centre the arcs subtending 3 and 4 of these parts. We thus obtain two base angles of the triangle and proceed as in (60).

26. (9) and (42). 27. (106). 28. Chapter X.

29. (127) to (130).

30. Find the angular points of a regular heptagon in the circle (101), and inscribe a circle in each sector as (139) *b*.

31. (61). 32. (5) and (42). 33. (46).

34. (127) to (130). 36. (5) and (47).

37. After drawing the rectangle **ABCD** join its diagonals **AC**, **BD** intersecting in **O**. Let **AB** be one of the longer sides. Place two straight lines parallel to **AB** and terminated by **OA**, **OB** of lengths 1½″ and 1″, (97) and complete the figures by drawing parallels to **BC** to meet **OC** and **OD** (98).

38. (46).

39. Let **P** and **Q** be the centres of the circles, where **B** lies on the circle, centre **Q**. Join **BQ** and produce it to meet, in **X**, the straight line through **A** bisecting **PQ** at right angles. Then **X** is the centre of the circle required, and **XB** its radius (see Chapter VII).

40. (7) and (85). 41. (38) or (41).

42. Let **ABCD** be the angular points of a square inscribed in the circle, found by drawing **AOC**, **BOD** two diameters at right angles. Bisect the arc **AB** in **D** and at **D** draw a tangent to the circle, meeting **OA** produced in **T**. Bisect the angle **DTO** by a straight line meeting **OD** in **X**. **X** is the centre of a circle, radius **XD**; in the sector **OAB**. The other three circles have their centres on the bisectors of the angles **BOC**, **COD**, **DOA** at a distance from **O** equal to **OX**, and the radius of each equal to **XD**.

43. The representative fraction $= \frac{1}{10}$, therefore we take $\frac{1}{10}$ of 5 feet = 6″. Divide the 6″ into 5 equal parts and the left-hand division into 12 to shew inches. Observe instructions given for plain scales in Chapter X.

44. Make the rectangle sides 3½″, 1½″; find its centre by joining its diagonals, describe a circle with this centre and radius ¾″. Draw a line through this centre parallel to the longest side of the rectangle.

Measure points ¾″ distant from each end, these are the centres of circles (**A** and **B** suppose). To find the radii of the smaller circles we observe that each has radius ¼ of **AB** that is ½″.

45. Draw a vertical straight line **AB** of length 1¼″. From **A** draw a horizontal line **AC** of ½″ and from **B** another **BD** = 1½″. Join **CD** by a fine or dotted line and bisect it in **E**; on **DE** on the side remote from **AB** describe an isosceles triangle having equal sides ¾″, and on **CE** towards **AB** make another equal isosceles triangle, in dotted lines. The vertices of these triangles are the centres for the arcs. Complete by drawing parallels to **AC** and **BD** ¼″ from them, as shewn in the figure.

46. (142).

47. Draw a horizontal straight line 1½″ long, divide it up into sections ¼″, ¼″, ½″, ¼″ and ¼″. On the middle division, ½″, describe an equilateral triangle and on each ¼″ at the end of the line describe half an equilateral triangle by drawing parallels to the sides of the centre one. Each parallelogram is a double equilateral triangle equal to the first, the sides of each being in the same straight line with the next one to it, and parallel to the sides of the first triangle, or in the same straight line with one of them. The angles of the half equilateral triangles mark points through which sides of the parallelograms would pass if produced.

48. The representative fraction is $\frac{8}{1200} = \frac{1}{150}$ therefore 90′ is shewn by $\frac{1}{150}$ of 90′ = 7·2″. [This length might also have been found by taking 9 times ·8 or by (39) as explained in Chapter X.] Divide 7·2″ into nine equal parts each to shew 10′ and divide the last division on the left into ten equal parts. Complete and figure as explained in Chapter X.

49. After constructing the first triangle (7), divide the straight line 8½″ in proportion 3, 2¼, 1¼ (38). With the parts thus obtained construct the triangle.

50. Compare with example 43. Divide 4¾″ into four equal parts and one of these into 12 equal parts.

51. After describing the first pentagon draw tangents to the circle at the middle points of the five equal arcs into which it divides the circle (45), (107) and (161).

52. Compare with example 37, taking **AB** now as a *shorter* side.

53. Draw two straight lines at angle 25° (see note to 26) at any point **P** in **AB** make a perpendicular **PQ**$=\frac{3}{4}''$, draw **QD** parallel to **AB** to meet **AC** in **D** the centre required. Describe the circle; a construction circle on **AD** as diameter will cut this circle where it touches **AB** and again at **E** the tangent point required. Join **AE**.

54. (127).

55. The part of the figure **CABE** has already been described with slightly different measurements. To complete the figure we have to construct the angles **AED** and **BCD** each 95° and produce the lines to meet. Compare figures (151), (152). Join **AD**, and draw a parallel through **E** to meet **BA** produced in **X**. Join **BD**, and draw a parallel through **C** to meet **AB** in **Y**. **DXY** is the triangle required (115).

56. (132).

57. This pattern has to be copied exactly from that given. The clue to the drawing is that the centres of the circles are the angular points of a regular hexagon, concentric and similarly situated with the outer ones. These centres are at the cusps of the foils and the distance between two adjacent ones can be measured from the pattern.

58. Divide the distance between the 10 and 40 into three equal parts, figure the divisions 20 and 30.

Produce the scale in the direction of the 10 for two divisions equal to those found above, figure the next division to the 10 as **O**, then divide the last division on the left-hand side into 5 equal parts, figure each division, reading from right to left 0, 2, 4, &c. up to 10. The scale may be produced to the right, and an equal division marking 50 be set off if required.

59. Draw a semicircle on a diameter $1\frac{1}{2}''$; find the angular points of a regular octagon by dividing the semicircle into four equal arcs. We do this by making a right angle at the centre and bisecting the arcs of the quadrants, or quarter circles thus found.

Take two adjacent angular points, **A** and **B** suppose. Make an isosceles triangle on **AB** of side $\frac{7}{8}''$ away from the centre. The vertex of this triangle is the centre of one of the outer arcs of radius $\frac{7}{8}''$. With the same centre and radius $1\frac{1}{8}''$ make an arc. This gives one of the inner arcs. To find the radius of the inner semicircle, join the centre of the first semicircle to the vertex of the isosceles triangle on **AB**, this line cuts the corresponding inner arc of radius $1\frac{1}{8}''$ at a point on the semicircle required, which has its centre at the centre of the first semicircle.

60. Draw the parallel lines; they form a parallelogram the sides of which are produced, four circles the centres of which are outside the parallelogram on the diagonals produced can be found as in (71).

61. Take a straight line **PQ**$=1\frac{1}{4}''$ and draw **APB**, **CQD** perpendicular to it at its ends. Measure distances **PL**, **PM** each $=\frac{1}{2}''$ along **AB**, in opposite directions; and **QX**, **XY** each $=\frac{13}{16}''$ along **CD**. The circle required passes through **LMXY**, therefore bisect **LX** at right angles by a straight line meeting **PQ** in **O**.

O is the centre required.

62. Draw perpendiculars to **OA**, **OB** at **A** and **B** to meet in **C**. With centre **C** distance **CA** or **CB** describe the circle. For the remaining tangent compare (72).

63. Make a right-angled triangle **ABC** right-angled at **A** where **BC**$=3''$ and **AB**$=1\frac{3}{4}''$ (27); then the equilateral triangle on **AC**$=$the difference of areas of those on $3''$ and $1\frac{3}{4}''$ [see notes on areas p. 78 to p. 81] the triangle can be reduced to a square (117).

64. (45). After joining diagonals circumscribe the figure by a circle by rules given in Chapter V.

65. (95).

66. First describe the triangle **ABC** for its sides are known, then the triangle **ABD**. Join **CD** and proceed to find a triangle of equal area (115) and then the square required (117).

67. Draw **OQ** perpendicular to **AB** where **OQ**$=1''$ to find **O** the centre of the first circle. Complete as in (74).

68. The straight line $3{\cdot}5''$ is divided into 3 and the left-hand division into 10 to shew single feet. Compare with example 48.

69. (8) and (117).

70. The R.F. for the new scale will be $1\frac{1}{2} \div 11$, therefore 4 feet will be shewn by $48''$ multiplied by this fraction$=6{\cdot}55''$ nearly. Divide the line and figure as in example 43.

71. The R.F.$=\frac{7}{10} \div [3280{\cdot}9 \times 12]$ see rule for R.F. in Chapter on Scales.

Take any distance, say 10 miles, reduce it to inches and multiply by the R.F. we get $3{\cdot}76''$ nearly. Now this distance is rather short for the scale required so double it to shew 20 miles. Bisect it, divide the left-hand division into 10 equal parts, shew furlongs diagonally by eight parallels. Compare with (110).

72. (54). 73. (72).

74. See article on Indices in Solid Geometry.

75. Draw the line XY (164), and make a square, ABCD, side $1\frac{1}{2}''$ one side of which AB is inclined to XY at an angle 32°. Draw projectors to XY and produce each $1\frac{1}{2}''$ beyond XY. Complete the rectangle, representing the edge out of sight by a dotted line. Now draw another ground line X′Y′ at angle 15° with XY and proceed as in (164). *The horizontal and vertical traces* of a plane are the intersections of the plane with the H.P. and V.P., and are therefore straight lines.

76. (127) or (128). Observe A and A′ are found by making $CA = CA' = BF_1$. CB and CF_1 being given as $1\frac{1}{8}''$ and $1''$.

77. This pattern is to be exactly copied. The centres of the circles form an equilateral triangle, and are found by (34).

78. Describe a square on a side $= 2\frac{1}{2}''$. Bisect each side. With centres at the angular points of the square and distance a bisection of an adjacent side describe arcs. The centre of the inner circle is at the centre of the square and it meets each of the quadrants just drawn where the diagonals of the square meet them.

79. See notes on Scales and figure (110). Draw to shew 10 yds. by 3″, divide the left-hand division into 3, and diagonally by 12 parallels.

80. This is a comparative scale; first find the R.F. by dividing 2·85″ by $3\frac{3}{4}$ versts reduced to inches, each verst = 42000″; then multiply a trial distance, say 10 miles, by the R.F. we get 3·82″. Complete as in example 71.

81. Make an equilateral triangle ABC sides 2″. With centres A, B, C radii 1″ make the three circles. The circumscribing circle will have its centre at O, the centre of the equilateral triangle ABC, found by joining two angles to the bisection of the opposite side, and the radius of the circle is found by joining OA and producing it to cut the circle centre A.

82. (73).

83. Draw two diameters at right angles to find the angular points of the square, step off the radius round the circle and take alternate points to find the equilateral triangle. Find the third proportional to the sides of the squares described in the question, these need not be measured, but as a test of work we find one side is 3″ and the other $\sqrt{6}''$, the third

proportional comes out exactly $2''$. Therefore the area of the square on the side of the triangle : area of the square in the circle : : 3 : 2 [see note to 157].

84. To describe the heptagon see Chapter IX, or the use of the sector, Chapter XV. Reduce it to triangle **ABC** of equal area, **BC** being the longest side, divide **BC** into three equal parts, **BD** being one of them. The triangle $\mathsf{ABD}=\frac{1}{3}$ area of heptagon. Make an equilateral triangle equal to **ABD**, (115) and (122).

85. Draw a rectangle $3\frac{1}{4}''$ by $2\frac{1}{4}''$. Find its centre **C** by joining its diagonals, draw dotted lines parallel to the sides of the rectangle and within it each at a distance $\frac{9}{16}''$ from the sides of the rectangle, the angular points of which are **A**, **B**, **D**, **E**. With centres **A**, **B**, **D**, **E** and radius $\frac{9}{16}''$ in each case describe four circles. **CA**, **CB**, **CD**, **CE**, dotted lines, cut these circles in points **P**, **Q**, **R**, **S**. A circle described, centre **C** and radius **CP**, should pass through **Q**, **R**, **S**.

86. Describe the arc of a circle with radius $2''$, along this radius from the centre mark off $1\frac{3}{8}''$, take this as centre of the second circle.

The centre of the smaller circle is found by describing arcs concentric with the first circles of radii $2''-\frac{5}{8}''=1\frac{3}{8}''$ and $1\frac{3}{8}''-\frac{5}{8}''=\frac{3}{4}''$. [See figures 67 to 69.]

87. The section is an equilateral triangle. The side of which is $1\frac{1}{6}''$ (171) to (173) and read note to example 123.

88. Produce **AC** to **E**. Make the angle $\mathsf{ECL}=45^\circ$. Bisect **LCA** by **CM** meeting **AB** in **M**. Through **M** draw **ML** parallel to **AC** meeting **CL** in **L**. Join **AL** meeting **BC** in **P**. Draw **PQ** parallel to **CL** to meet **AC**, and **PR** parallel to **LM** to meet **AB**. Measure **QS** along **QA** equal to **PR**. Join **RS**. **PQSR** is the figure required for the angles **LCM** and **LMC** each$=67\frac{1}{2}^\circ$. Therefore $\mathsf{LC}=\mathsf{LM}$ and **A** is a centre of similitude for the figures **AQPR** and **ACLM**. Therefore $\mathsf{PQ}=\mathsf{PR}=\mathsf{QS}=\mathsf{RS}$ and $\mathsf{RSQ}=\mathsf{PQC}=\mathsf{LCE}=45^\circ$.

89. Draw a circle, centre **O**, diameter $\mathsf{COD}=2{\cdot}7''$. Draw any tangent **PQ** at any point $\mathsf{P}=2{\cdot}25''$. With centre **O**, and distance **OQ** make an arc **QT** to meet **CD** in **T**, then $\mathsf{CT}=x=3{\cdot}36\ldots.$ **DT** is very nearly exactly $\frac{2}{3}''$, and $\mathsf{CT}=\frac{2}{3}''+2{\cdot}7''$.

90. (27). Place the squares on the two sides of the triangle side by side to form a single figure, reduce this figure to a triangle of equal area, and the triangle to a square (115) and (117). Measure its side, which should be $2''$, and write its length upon it.

91. Let **ABCD** be the square, angles in order of the letters. Measure **AP** and **CQ** each$=1''$, along **AB** and **CD**. **PQ** bisects the square. Through **C** draw a parallel to **BQ** to meet **AB** produced in **T**, and through **A** draw a parallel to **PD** to meet **CD** produced in **S**. **PTQS** is the parallelogram required.

92. Draw perpendiculars **PA**$=\frac{3}{4}''$ and **QB**$=1''$ at **A** and **B** to **AB** to find the centres of the circle. Join **PQ** and make the angles **CPQ** and **DQP** equal to **APQ** and **BQP** on the other side of **PQ** to find the other points of contact.

93. (129). 94. (5) and (117).

95. Compare (109) and examples 13 and 36.

In the second part of the question $1{\cdot}15''$ represent ten paces $=340$ inches, therefore his R.F.$=1{\cdot}15\div340$ and the scale is taken to represent 100 yards with this R.F.

96. (108).

97. Find the angular points of a regular hexagon inscribed in the circle by stepping off the radius on the circumference. Join these angular points to the centre and in each of the equal sectors thus obtained inscribe a circle. A circle is inscribed in a sector by drawing a tangent at the middle point of the arc, producing the radii containing it to meet the tangent, and inscribe a circle in the isosceles triangle thus obtained.

Compare figure (139) *b*.

98. (95).

99. Reduce the irregular figure to a triangle of equal area and proceed as in (121) observing notes at beginning of Chapter XII.

100. Take a central angle of 40° to find a side of the nonagon, or use (103) or (104).

Draw a diameter of the circle parallel to a side of the nonagon and measure $\frac{3}{4}''$ both ways along this diameter from the centre. Through the points thus obtained draw perpendiculars to the diameter; they will cut the nonagon in the angular points of the rectangle required.

101. Take 10 yds. shewn by $\frac{5}{4}$ of $3\frac{1}{2}''$. Find a fourth proportional to 4, 5, $3\frac{1}{2}$ by (39). Result$=4\frac{3}{8}''$ and complete as in example 68.

102. (106).

103. Let ABCD be the square at A, make BAM = 60° and at B make ABM = 70°. Through D draw a parallel to AM, and through C a parallel to BM, these straight lines meet each other and AB produced both ways in the angular points of the triangle required.

104. Describe a rectangle 2½″ by 2¾″ and draw parallels to its sides as shewn within, always at ¼″ distance from the last drawn.

105. Describe a regular hexagon of side 1¼″ [page 10]. Bisect its sides, and again bisect the sides of the inner figure obtained by joining the points of bisection.

106. Describe a circle of radius 1½″. Find the angular points of a regular hexagon in it [page 10]. Describe a dotted concentric circle of radius 1″, and join the angular points of the hexagon to the centre. These lines cut the dotted circle at the centres of the arcs forming the foils.

The inner circle has radius ⅜″.

107. Compare examples 68 and 101.

108. Describe the square, and join its diagonals, from the centre O along a diagonal AC measure ½″ both ways to points P and Q. Through P and Q draw perpendiculars to AC, these meet the sides of the square in the angular points of the rectangle.

109. Let ABC be the triangle, AB = 2⅛″, BC = 1⅞″, CA = 1½″ (7). Produce AB to D making AD = 2½″. Through D draw a parallel to BC to meet AC produced in E. DAE is the triangle required [see Chapter VIII].

110. Find the angular points of the heptagon by (101), and complete figure as shewn.

111. Bisect the angle BAC by AX passing through O the centre of the first circle which is found as in (71) and let AO produced cut the circle in P. At P draw XY at right angles to AO meeting AB and AC in X and Y. Bisect the angle BOY by OZ meeting AX in Z. Z is the centre of the second circle, radius ZP. AZ produced meets this second circle in a point Q. Draw a perpendicular to AQ through Q to meet AB, and AC and proceed as before.

112. (164).

113. Find the centre O of an equilateral triangle ABC by joining the angles to the middle points of the opposite sides. D the middle point BC; E, of AC; F, of AB. In each of the figures AEOF, CEOD, BDOF inscribe a circle (50).

114. Draw a straight line $2''$ long as shewn by the figure. Mark off $\frac{1}{4}''$ from each end and at the points thus found draw perpendiculars, the lower one $1\frac{1}{2}''$, the upper one $1''$. At the ends of these perpendiculars draw dotted parallels to the $2''$ of lengths $1''$ and $\frac{1}{2}''$ respectively. The ends of these lines are the centres of the quadrants required. Complete the figure as shewn.

115. (146).

116. This figure can be drawn to the dimensions shewn very easily till we get to the last arc of a circle (not the quadrant). To find the centre of this arc join the two points on it which are known, namely the lowest point of the quadrant, A suppose, and B the extremity of the upper of the lower parallels of $\frac{5}{8}''$. Join A, B by a dotted line. Bisect AB at right angles by a straight line (dotted) meeting the horizontal through A in C.

C is the centre of the lower arc and CA or CB its radius.

117. Draw half an ellipse of major axis $3''$ and half minor axis $1''$. Compare with (140).

118. Compare with (173).

119. Compare with (169), (173) and (174).

120. Draw the straight line XY. Below it make a rectangle ABCD, $AB = CD = 1''$ parallel to XY, and BC, DA in a direction perpendicular to it, each $= 2''$. BC and DA produced (in dotted lines) meet XY in L and M so that $LM = 1''$. On LM describe the elevation of the prism, a triangle LMN having sides $LM = 1''$, $ML = 1\frac{1}{4}''$, $MN = 1\frac{1}{2}''$. From N draw a dotted line NP perpendicular to XY, and produce it to meet AB and CD in E and F. Draw EF. EF is not dotted, but rather a thick line being part of the plan, which is completed by the rest of the rectangle ABCD.

121. Compare with (164).

122. Construct two circles one above XY, centre C, and another, centre D, vertically below XY each with radius $1''$. Draw a dotted line CD produced both ways. On CD take a point $\frac{5}{8}''$ above C and draw through it a parallel to XY to cut the circle in L and M, and rub out the portion of the circle above LM. On CD take a point $\frac{1}{2}''$ below D, and draw through it a parallel to XY to meet the circle in P and Q. Rub out the portion of the circle below PQ. Through L and M at right angles to XY draw dotted lines to meet PQ in E and F. With centre D make an arc of a circle at distance DE above EF, terminated by E and F. Shade in this inner circle.

123. Compare with (169), observing that a section of a plan is obtained from an elevation, exactly as an elevation is obtained from a plan, that is draw perpendiculars from the first elevation and measure distances on the first plan for the second.

124. Draw perpendiculars (dotted) from all angular points on the plan, when copied to right scale, and produce them above the line *xy*.

The angular points of the lower bricks will be respectively on the line *xy* and at a height of ½″, two angular points on each projector; the edges of the bricks not in sight should be dotted.

The angular points of the brick resting on the other two are ½″ and 1″ above the line *xy*, two on each projector. Join all edges; dotting those not in sight.

125. Draw the rectangle of sides 2″ and 3″. Bisect the shorter sides and draw lines making 45° with them and produce to cut the longer sides. These lines give two sides of each of the larger squares, which can be completed.

To get the smaller squares we need only observe that they have their diagonals in the same straight lines with the larger ones, an angular point of each of the smaller squares being at the intersection of the diagonals of the rectangle. Beginning with this point draw parallels to the sides of the larger squares, to meet their diagonals.

126. Let ABC be the triangle, AB = 1½″, BC = 2″, CA = 2½″.

Through A draw AFG parallel to BC. Produce BA to D so that BD = BA + AC, and bisect it at E. With centre B make an arc EF to meet AFG in F. Bisect BC in K. Draw KG parallel to BF. Then BFGK is the figure required for BK + FG = BK + KC = BC.

BF + KG = 2BF = 2BE = BD = BA + AC and the parallelogram is equal in area to the triangle for it is the same height on half the base.

127. Find the angular points of a regular pentagon in the circle. Join the centre to each of the angular points. Inscribe a circle in each of the sectors thus formed. See (45) and compare example 97 and (139) *b*.

128. Take AB = 2″, BC = 2½″, CA = 3″. Bisect the angle B by a straight line meeting CA in D. D is equally distant from BA and BC.

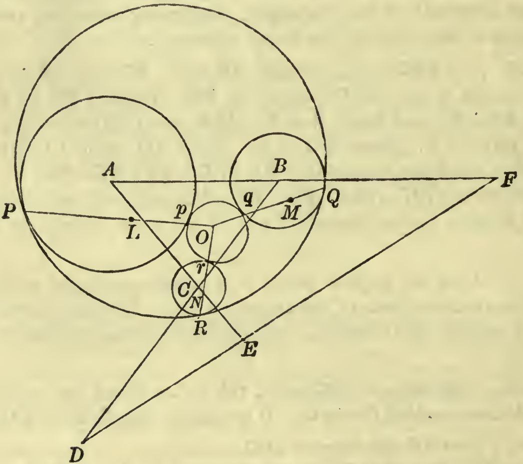
129. Join AB and bisect it at D.

Join CD, then CD is equally distant from A and B, for drawing AM and BN perpendicular to CD we find BND and AMD are equal right-angled triangles.

130. Compare with example 108.

131. There is an approximate method for finding the centres required given in Chapter XV, but as it may be of interest to know that the circles can be exactly obtained by a method invented by Gergonne (Euclid I—VI by H. M. Taylor, p. 458) it is here quoted. Expressions are used in it that need explanation. When two tangents to a circle at **X** and **Y** intersect in **Z**, **Z** is called the pole of the straight line **XY**, and **XY** is called the polar of **Z**. If **O** be the centre of the circle and **OZ** cuts **XY** in **H** and the circle in **K OZ : OK :: OK : OH**. But if the point **Z** were within the circle so that two tangents from **Z** cannot be drawn to it, then join **OZ**, produce it to **H** so that **OZ : OK :: OK : OH** where **OK** is the radius, and draw **XY** through **H** at right angles to **OH**. Now **XY** is the polar of the pole **Z**, and the tangents at the extremities of any chord through **Z** intersect on **XY**.

The points at which the common tangents to two circles intersect are centres of similitude (65) and (66). When there are three circles we get six centres of similitude found by taking them by pairs, these lie three by three on four straight lines which are called axes of similitude.



When two circles intersect, their common chord is called the radical axis, and even when they do not intersect there is a radical axis which is the straight line from which equal tangents can be drawn to both circles, found by drawing common tangents to both circles and joining their points of bisection. When there are three circles, we have three radical axes found by taking the circles by pairs, these intersect at a point called the radical centre.

The construction is then as follows:—

"Let **A**, **B**, **C** be the centres of three given circles. Find **O** the radical centre and draw **DEF** one of the axes of similitude of the three circles.

"Find poles **L**, **M**, **N** of **DEF** with respect to the three circles, centres **A**, **B**, **C**. **OL**, **OM**, **ON** cut the circles in **P**, p; **Q**, q; **R**, r.

"The required circles pass through **P**, **Q**, **R** and p, q, r."

It is not necessary to find all the centres of similitude, for if two of different pairs of circles are taken, **DEF** can be drawn through them.

132. Let **BC** be the straight line. On it describe an equilateral triangle **ABC**.

Bisect the angles **ABC** and **ACB** by straight lines meeting at **O**. Draw parallels through **O** to **AB** and **AC** meeting **BC** in **D** and **E**, then **BD**=**DE**=**EC**.

133. This example is introduced to shew an interesting way of drawing tangents to any circle, or conic section given under the heading 'Geometry' in the *Encyclopædia Britannica*.

Let **P** be the external point, draw any two straight lines through **P** to meet the conic in **B**, **C**, and **D**, **E**. Let the chords **BE** and **CD** meet in **F**. Draw a straight line through **P** and **F** to meet the curve in **G** and **H**. Let the chords **GD** and **HE** meet in **K**.

KF meets the curve in **M** and **L**, the points at which straight lines drawn from **P** touch it. By this construction tangents can be drawn by the ruler and pencil only. Another method more easy to remember is this:—

Let **P** be the external point, draw any two straight lines through **P** to meet the curve in **B**, **C** and **D**, **E**; draw tangents at **B**, **C** and **D**, **E** to meet respectively in **X** and **Y** (130). A straight line drawn through **X** and **Y** meets the curve in **M** and **L**, the points required.

The proof depends upon a property of pole and polar defined in the solution of example 131. The polars of **X** and **Y** pass through **O**, therefore the polar of **O** passes through **X** and **Y**.

134. The method of describing these angles is given in Chapter II.

Describe a semicircle **ABCDE** on any straight line **AE** having centre **O**. Draw **OC** perpendicular to **AE** to meet the semicircle in **C**, and step off the radius **OA** round the semicircle meeting it in **B** and **D**.

Then the arcs BC, CD each subtend 30° at the centre. Bisect the arc BC at F and divide the arc BF by trial into three equal parts, the point of division nearest to F being H. Then the angle AOH = 70° being 75° less by one-third part of 50°.

95° is obtained by setting off an arc = HF on the circle from E beyond BE, EK suppose, then AOK = 95°.

The angle AOD = 120°, so measuring an arc = BH from D towards E, DM suppose, we get the angle AOM = 110°.

Bisect the angle COE by ON then AON = 135°.

135. Make AB = 2″. Bisect it at right angles by CD. Produce BA to P and make the angle PAQ = 100°. Bisect the angle QAB by AD meeting CD in D. Join DB, then AD = DB and angle ADB = 100°. The remaining triangles are described in the same way.

136. Make the angle BAC = 100° [see Chapter II] and AB = AC = 2″. With centre A make the arc BC. Bisect the arc BC at D by a straight line bisecting the angle BAC. Draw the tangent at D, which is either parallel to the straight line BC, or at right angles to AD and let it meet AB produced in T. The straight line bisecting the angle ATD meets OD in X. Then X is the centre of the circle required and XD its radius.

137. See Chapter XIII.

$\sqrt{3}$ is either the mean proportional between 1 and 3, or a side of a right-angled triangle of hypotenuse 2 and the other side 1.

$\sqrt{\tfrac{1}{3}}$ is a mean proportional between 1 and $\tfrac{1}{3}$, or a third proportional to $\sqrt{3}$ and 1.

$\dfrac{2}{\sqrt{3}}$ is a fourth proportional to 2, $\sqrt{3}$, and 1, or else may be taken as double $\sqrt{\tfrac{1}{3}}$ found above.

138. One method of solution has been already given [page 136]. Another solution is as follows. Describe a triangle the base of which is 5″ being the sum of the sides, its height 2″, and its vertical angle = 90° + half the given angle = 110° (64). Let AB be the base of this triangle = 5″ and C its vertical angle. Bisect CA and CB at right angles by straight lines PQ and LM, meeting the base AB in Q and M. Join CQ, CM, then CMQ is the triangle required. For first it is evident that AQ = CQ and BM = CM, therefore AB = CQ + QM + CM. Next the angle CQM = 2CAB and CMQ = 2CBA, and CAB + CBA + 110° = 180°, therefore CAB + CBA = 70°, therefore CQM + CMQ = 140°, so that QCM = 40°.

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